Answer THREE questions with at least ONE from each Section.

Section A. Answer at least ONE question

1. a. i. A binary search tree with n nodes can be represented by four n-size arrays. What are these four arrays?

   ii. Give a definition for the binary search tree property.

   iii. Give a definition for the heap property.

   iv. Why is a binary search tree (with the binary search tree property holding) a better data structure for random access than a heap (with the heap property holding)?

[8 marks]

b. Draw the heap that corresponds to the following array

$$\langle 4, 1, 3, 2, 16, 9, 10, 14, 8, 7 \rangle$$

Explain how a sequence of applications of heapify on this heap can give a heap with the heap property holding. Also, give the array corresponding to this new heap.

[8 marks]

c. i. What is a trie and why is it useful for text coding?

   ii. Draw the highest trie that can be formed with four leaves, and then give a string of 12 characters that when Huffman's algorithm is applied gives this trie.

   iii. Draw the lowest trie that can be formed with four leaves, and then give a string of 12 characters that when Huffman's algorithm is applied gives this trie.

[9 marks]

d. Explain how Huffman's algorithm can be generalized to ternary codewords (ie.code words using the symbols 0,1, and 2). Also explain why it gives an optimal coding of the input text.

[8 marks]

[Total 33 marks]

2. See 2b12.q2.ps

CONTINUED

3. See 2b12.q3.ps

TURN OVER

Section B. Answer at least ONE question

4. a. Let L be a first-order language, $M = (D, I)$ be an L-structure and let ϕ be a formula of L.

   i. Say what it means for ϕ to be *valid* in M.

   ii. Say what it means for ϕ to be *satisfiable* in M.

[8 marks]

b. Let ϕ be any formula and let M be a structure. Is the following statement true?

$$\neg\phi \text{ is satisfiable in } M \iff M \not\models \phi$$

If you think the statement is true then explain why; if not then give a counter-example.

[10 marks]

c. Let L be a first-order language with constant symbol $C = \{z\}$, function symbols $F = \{\}$ and predicate symbols $P = \{eq^2, g^2\}$. Let Z be the structure $(\mathbb{Z}, I)$ where $\mathbb{Z}$ is the set of integers, $I = (I_c, I_f, I_p)$ and

$$\begin{aligned} I_c(z) &= 0 \\ I_p(eq^2) &= \{(n, n) : n \in \mathbb{Z}\} \\ I_p(g^2) &= \{(m, n) : m > n\} \end{aligned}$$

For each formula below say if the formula is valid in Z and if it is satisfiable in Z. If the formula is not valid in Z, give an assignment which falsifies the formula. If the formula is satisfiable in Z, give an assignment that satisfies it.

   i. $\neg((g^2(x, y) \vee g^2(y, x)) \vee eq^2(x, y))$

   ii. $\forall x \forall y((g^2(x, y) \vee g^2(y, x)) \vee eq^2(x, y))$

   iii. $(g^2(x, y) \to \exists z(g^2(x, z) \wedge g^2(z, y)))$.

   iv. $(g^2(x, z) \wedge \forall y(g^2(y, z) \to (g^2(y, x) \vee eq^2(y, x))))$.

[15 marks]

[Total 33 marks]

5. a. What does it mean when we say that a first-order formula is *valid*?

[5 marks]

b. Explain how to use a tableau to discover if a first-order formula is valid or not.

[12 marks]

c. For each of the following formulas, construct an appropriate tableau and use this to determine whether the formula is valid.

i. $\exists x \forall y C^2(x, y) \to \forall y \exists x C^2(x, y)$

ii. $\exists x (P^1(x) \vee Q^1(x)) \to (\exists x P^1(x) \vee \exists x Q^1(x))$

iii. $\neg[(\exists x \exists y (x < y)) \wedge \forall x \forall y (x < y \to \exists z (x < z \wedge z < y))]$

[16 marks]

[Total 33 marks]

END OF PAPER