Answer THREE questions with at least ONE from each Section.

Section 1..Answer at least ONE question

1. a. Consider depth-first search.

   i. Give a recursive definition.

   ii. Give a definition based on a stack.

[13 marks]

b. A binary tree can be represented as a nested list, where a list is a triple containing a node and a list for each branch.

   i. Draw the binary tree corresponding to the following nested list:

$$[a,[b,[d,[],[]],[e,[g,[],[]],[]]],[c,[],[f,[h,[],[]],[i,[],[]]]]].$$

   ii. A nested list, such as in (i), was constructed using pre-order traversal. Construct another list for the same tree using in-order traversal.

   iii. Represent the tree in (i) as a directed graph (N, A), where N is a set of nodes and A is a set of directed arcs.

[15 marks]

c. For the list of part b(i), give the sequence of nodes visited in a depth-first search and in a breadth-first search.

[5 marks]

[Total 33 marks]

2. a. Compare and contrast the following approaches to designing algorithms:

   i. Greedy algorthms.

   ii. Divide and conquer.

   iii. Dynamic programming.

[15 marks]

b. Consider Dijkstra's algorithm.

   i. What are the conditions on the all input to the algorithm?

   ii. What is the output from the algorithm?

   iii. Give an example of input to the algorithm that has five nodes and ten arcs, and give the output from the algorithm.

   iv. Explain in what way the algorithm is a greedy algorithm.

[10 marks]

c. Explain why there is no greedy algorithm that is guaranteed to give an optimal solution for the travelling salesperson problem.

[8 marks]

[Total 33 marks]

3. a. i. Explain briefly the *greedy approach* to algorithms.

ii. Explain briefly how Kruskal's algorithm can be used to find a minimum spanning tree.

[10 marks]

b. A coin system is a set of coins of different integer values, $c_1, .., c_n$, where $1 < c_1 < ... < c_n$. For example, the Britsh system is the set $\{1, 2, 5, 10, 20, 50, 100\}$. Consider the problem of giving change for a total value X using this currency system, so that the least number of coins is used. In other words, we need to find integers $k_1, k_2, .., k_n$ such that:

$$k_1 c_1 + ... + k_n c_n = X$$

and $k_1 + ... + k_n$ is minimized.

i. Give the psudocode for a greedy algorithm that solves this problem for a wide variety of coin systems.

ii. Show that the algorithm works for the British coin system for giving change for 276p.

iii. Give a coin system for which your algorithm does not work. Explain why it does not work.

[23 marks]

[Total 33 marks]

Section 2..Answer at least ONE question

4. This question is about propositional logic.

   a. Find equivalents to each of the formulas below in a propositional language using negation ($\neg$) and conjunction ($\wedge$) only.

      1. $(p \rightarrow q)$
      2. $(p \vee q)$
      3. $(p \rightarrow \neg(q \vee \neg r))$

   [9 marks]

   b. Let $\perp$ be a propositional constant that always evaluates to false. Find equivalents to each of the formulas below in a propositional language using implication ($\rightarrow$) and $\perp$ only.

      1. $\neg p$
      2. $\neg(p \vee q)$
      3. $(p \vee \neg(q \wedge \neg r))$

   [9 marks]

   c. For each of the formulas below construct a complete tableau with the formula at the root. Use the tableau to determine whether the formula is satisfiable or not. If the formula is satisfiable, use your tableau to find a valuation that satisfies the formula.

      1. $\neg(p \rightarrow (q \rightarrow p))$
      2. $\neg((p \rightarrow q) \rightarrow p)$
      3. $\neg(q \rightarrow (\neg p \wedge (q \rightarrow p)))$.

   [15 marks]

   [Total 33 marks]

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5. This question is about Boolean algebra.

a. State De Morgan's laws and demonstrate their validity using a truth table.

[6 marks]

b. Explain how a Karnaugh Map can be used to simplify a Boolean expression.

[6 marks]

c. Simplify the expression

$$\bar{a}.\bar{b}.c.\bar{d} + \bar{a}.\bar{b}.c.d + \bar{a}.b.c.d + \bar{a}.b.\bar{c}.d + a.\bar{b}.c.d + a.b.\bar{c}.d$$

[6 marks]

d. How can a Karnaugh Map be used to find the conjunctive normal form of an expression? Find the conjunctive normal form of the expression given in the previous part of this question.

[15 marks]

[Total = 33 marks]

6. Let A consist of all instances of the axiom schemes

- $(\phi \to \neg\neg\phi)$
- $(\theta \to (\phi \to \theta))$
- $((\theta \to (\phi \to \psi)) \to ((\theta \to \psi) \to (\theta \to \psi)))$
- $((\neg\theta \to \neg\phi) \to (\phi \to \theta))$

a. Define the 'provability relation' $\vdash$ in terms of A, stating clearly which inference rule(s) are to be used.

[9 marks]

b. Explain what we mean when we say that the provability relation $\vdash$ (from previous part) is (i) sound and (ii) complete for the propositional logic of formulas using the connectives $\neg$ and $\to$ only.

[8 marks]

c. Using the axiom schemes A together with your chosen inference rule(s) give a proof of the formula $(p \to p)$.

[16 marks]

[Total = 33 marks]

END OF PAPER