Answer THREE questions with at least ONE from each Section.

Section 1..Answer at least ONE question

1. a. i. What is an articulation point in a graph?

   ii. Give a graph of 8 nodes and 8 arcs that has 2 articulation points.

[5 marks]

b. i. If a tree has n nodes, how many edges does it have? Explain your answer?

   ii. What is a binary tree?

   iii. What is the minimum and maximum height of a binary tree with n nodes? Explain you answer.

   iv. What is a complete binary tree?

   v. What is the minimum and maximum number of leaves in a complete binary treeof height h? Explain you answer.

[18 marks]

c. i. What is a heap?

   ii. Explain how to represent any array as a heap so that the orignal array can be recovered from the heap.

   iii. Draw a heap that corresponds to the array $[7, 8, 2, 3, 5, 6, 8]$.

   iv. For the heap generated in part (iii), write down the sequence of nodes visited in a depth-first search and in a breadth-first search.

[10 marks]

[Total 33 marks]

2. a. In order to adopt a divide and conquer approach for an algorithmic problem, give three requirements of the problem.

[6 marks]

b. Briefly explain how the following algorithms work and in particular explain how they adopt the divide and conquer approach:

i. MergeSort

ii. Quicksort

[10 marks]

c. i. Explain how and why the QuickSort algorithm would use the InsertionSort algorithm.

[2 marks]

ii. Give an example of an array containing 9 items that would give worst case performance by the InsertionSort algorithm. Explain how the inefficiency would arise.

[4 marks]

iii. Explain how a call to the pivot procedure of Quicksort works on the subarray [5,3,7,2,1], assuming the usual ordering on the natural numbers. Give details of the pointers used in the procedure.

[11 marks]

[Total 33 marks]

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3. a. i. Describe the features of an algorithm that would allow it to be classified as a dynamic programming algorithm.

ii. Why is dynamic programming a useful approach to designing algorithms?

iii. Give a disadvantage of the dynamic programming approach.

[13 marks]

b. Consider the following adjacency matrix for a graph with the nodes $\{a, b, c, d, e, f\}$.

	a	b	c	d	e	f
a	0	7	∞	∞	4	∞
b	2	0	7	∞	∞	∞
c	∞	∞	0	∞	∞	3
d	∞	∞	∞	0	8	∞
e	∞	∞	∞	∞	0	1
f	∞	8	∞	∞	∞	∞

i. Draw a graph corresponding to this adjacency matrix.

ii. Using the graph generated in part (i), give the input matrix for Warshall's algorithm.

iii. Using the input matrix generated in part (ii), give the intermediate matrices and final matrix that would be generated by Warshall's algorithm. Ensure that you present your answer clearly.

iv. Using the final matrix generated in part (iii), explain the relationships between the nodes $\{a, b, c, d, e, f\}$.

[20 marks]

[Total 33 marks]

Section 2..Answer at least ONE question

4. This question is about propositional logic.

   a. Explain what is meant by *disjunctive normal form*.

   [4 marks]

   b. Which of the following formulas are satisfiable?

   1. $(p \to \neg p)$
   2. $\neg(p \to (q \to p))$
   3. $\neg((p \to q) \to p)$
   4. $(((p \wedge q) \to r) \wedge \neg(p \to (q \to r)))$

   [8 marks]

   c. Which of the following formulas are valid?

   1. $(p \to \neg p)$
   2. $((p \to q) \to (q \to p))$
   3. $((p \to q) \to (\neg q \to \neg p))$
   4. $(p \to (q \to r)) \to ((p \wedge q) \to r)$.

   [8 marks]

   d. Let $\phi = (((p \vee q) \to p) \to (p \wedge \neg q))$. Construct a complete tableau with ϕ at the root. Use your tableau to determine if ϕ is satisfiable or not. Also, use your tableau to find a formula in disjunctive normal form, equivalent to ϕ.

   [13 marks]

[Total = 33 marks]

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5. This question is about boolean algebra.

   a. Define the NOR operator, $*$, for boolean algebra, by drawing a truth table for it.

   [2 marks]

   b. Find an expressions equivalent to each of the following using the NOR operator only.

      1. $-a$
      2. $a.b$
      3. $a + b$
      4. $a.\bar{b}$

   [8 marks]

   c. Write each of the expressions below in conjunctive normal form, i.e. as a product of sums.

      1. $a.b + \bar{a}.\bar{b}$.
      2. $\bar{a}.b.c + a.b.\bar{c}$

   [10 marks]

   d. For each of the following expressions (i) write the expression as a sum of products (disjunctive normal form) (ii) draw a Karnaugh map for the expression and (iii) use your Karnaugh map to simplify the expression, if possible.

      1. $(-(\bar{a}.\bar{b}).\bar{b}) + -(\bar{a} + \bar{b})$.
      2. $(\bar{a} + \bar{b} + \bar{c}).(a + \bar{b} + \bar{c}).(\bar{a} + b + c).(a + b + c)$.

   [13 marks]

[Total = 33 marks]

6. a. What does it mean when we say that two propositional formulas are *semantically equivalent* to each other?

[5 marks]

b. Let ϕ be an arbitrary propositional formula using any of the connectives $\neg, \vee, \wedge, \rightarrow$. Explain how it is possible to find a propositional formula ϕ^*, semantically equivalent to ϕ, and using only the connectives $\neg, \rightarrow$.

[7 marks]

c. What does it mean when we say that a proof system $\vdash$ is sound and complete for a propositional language?

[8 marks]

Let L be the propositional language of all formulas using only the connectives $\neg$ and $\rightarrow$ and let L^+ be the propositional language of all formulas using only the connectives $\neg, \vee, \wedge, \rightarrow$.

d. Let $\vdash$ be a sound a complete proof system for L using axioms A. Give some additional axiom schemes that can be added to A in order to make the proof system sound and complete for L^+.

[11 marks]

END OF PAPER