

**University College London**

*University of London*

**EXAMINATION FOR INTERNAL STUDENTS**

*For the following qualifications :-*

M.Res

COURSE CODE : **nDSP**

TITLE OF EXAMINATION : **n-Dimensional Signal Processing**

DATE : **2-March-2001**

TIME : **14:00**

TIME ALLOWED : **2 hours 30 minutes**

**TURN OVER**

*Answer 3 questions*

1.
  - a Where  $\mathbf{y} = \mathbf{Ax}$ ,  $\mathbf{A}$  being an  $m \times n$  matrix, distinguish between over and under specified systems. What is the Rank of  $\mathbf{A}$ ? Indicate what are null functions in such a system of equations. [5 marks]
  - b When solving an inverse problem by an iterative method, why must the form of the forward operator be well established? Suggest some methods for accelerating the convergence of iterative methods, and indicate some stopping rules. [10 marks]
  - c How may the Moore-Penrose Inverse (MPI) be determined by use of the Singular Value Decomposition (SVD)? What properties does it, the MPI, have? [5 marks]
  - d What is meant by the spectrum of singular values? Where singular values are very small, what techniques can be used to stabilise or regularise the solution? [10 marks]
  
2.
  - a What is the Fourier transform of a comb function used to sample in time with a spacing of  $\Delta t$ ? Draw a diagram, indicating the Nyquist and sampling frequencies. [6 marks]
  - b What is aliasing and why does it occur when acquiring data digitally?. How can it be avoided or removed, in particular when acquiring a sequence of images separated in time? [8 marks]
  - c To reconstruct correctly the original continuous data from sampled data, in three dimensions, what kind of interpolation should be used? If tri-linear interpolation is used instead, estimate the error. [8 marks]
  - d What are quantisation errors? What effects might be seen on the histogram of pixel values? Indicate some methods for reducing the effects of such errors. [8 marks]

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3. a What defines the theoretical limit for lossless data compression? Estimate a value for this limit for a 24bit (RGB) colour photograph, in bits per pixel? [6 marks]
- b Describe in reasonable detail the Lempel-Ziv data compression algorithm, indicating its potential advantages or disadvantages, and in particular comment on its speed and complexity with respect to Huffman coding. [8 marks]
- c Describe in reasonable detail the JPEG data compression algorithm. Why is the Discrete Cosine Transform (DCT) used? Compare the speed and performance of the JPEG algorithm to that obtained by using the DCT transform for a whole image without dividing the image into smaller blocks. [8 marks]
- d What kinds of differences might you expect with an algorithm such as JPEG using the DCT transform to that using a wavelet transform (such as JPEG2000)? How would you compare and measure their performance? [8 marks]
4. a Define firstly, the Covariance matrix derived from the pixels of an image, and secondly, the Co-occurrence matrix. [6 marks]
- b Describe what is meant by an affine image registration, and indicate the form of the transform to map pixels in 3D space between two image data sets. [8 marks]
- c What is the difference between rigid body and non-rigid body registration? Describe in reasonable detail how you would add constraints to the solution of a non-rigid body registration. [10 marks]
- d How could you use the Co-occurrence matrix to estimate texture in an image? Suggest an alternative method of estimating texture. [6 marks]

TURN OVER

5. a What is meant by a scale-space analysis of an image ? For what applications is it advantageous to use such an analysis for extracting information from images ? [5 marks]
- b A general form of nonlinear diffusion scheme of an image  $I(\mathbf{r})$  can be represented

Install Equation Editor and double-click here to view equation.

as a Partial Differential Equation:

where the initial state  $f(\mathbf{r}, t=0)$  is set equal to  $I(\mathbf{r})$ . Show that if  $g(\mathbf{r}, t)$  is a constant  $c$  then the resultant image at any subsequent time  $f(\mathbf{r}, t=t_1)$  is the same as convolution with a Gaussian of width  $\sigma^2 = 2 c t_1$ . [4 marks]

- c In the Perona-Malik diffusion scheme, the function  $g(\mathbf{r}, t)$  is made dependent on the image gradient at each point. Discuss appropriate forms of the function  $g(\mathbf{r}, t)$  and how they give rise to different evolution schemes for the image. [8 marks]
- d Describe a Finite Difference implementation of the Perona-Malik diffusion scheme, and discuss the advantage of Implicit, Explicit or Alternating Direction Implicit methods for its solution. [8 marks]

[END OF PAPER]