University College London

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

M.Res

COURSE CODE	:	**MMAI**
TITLE OF EXAMINATION	:	**Mathematical Methods, Algorithms and Implementations**
DATE	:	**4-March-1998**
TIME	:	**09.30**
TIME ALLOWED	:	**2 hours**

TURN OVER

Answer two questions

1) a) Define the operation of *convolution* of two one dimensional signals, in both its continuous and discrete forms. State the convolution theorem

[5 Marks]

b) Suppose you are given a list $\{f_1, f_2,...f_N\}$ of N samples of a function f(t) taken at equal intervals Δt from t = -T to T. This list is passed to a Discrete Fourier Transform (DFT) to produce another list $\{F_1,F_2,...F_N\}$. If the second list is assumed to be samples of another function F(w), then state the range and interval of sampling of the parameter w.

[4 Marks]

c) If $g(t;s) = (2\pi\ s^2)^{-1/2}Exp(-t^2/(2\ s^2))$ is a Gaussian of width s, and the convolution of f(t) with g(t;s) is denoted h(t;s) then show how to find samples of {h} of h(t;s) by muliplication of the list {F} with another list {G} and using an inverse Fourier Transform. Show how to find {G}

i) by DFT of a list {g}

ii) directly by sampling function G(w,s)

Discuss what possible advantages either method i) or ii) might have over the other.

[8 Marks]

d) Describe how to obtain the list {h} by a diffusion process acting on {f}. Include in your answer an explanation of the *implicit* and the *explicit* procedure for implementing the diffusion process, and describe any advantages one might have over the other.

[8 Marks]

[**CONTINUED**]

2) a) Consider the problem

$$u''(x) = K\, u(x)$$

with $u(0) = u(L) = 0$

show that $u_n(x) = \mathrm{Sin}(n\,\pi\, x/L)$ are solutions to this problem for any positive integer n.

[4 Marks]

b) How can the answer to part a) be used to find the solution to the problem:

$$u''(x) - \alpha\, u(x) = f(x)$$

where $f(x)$ is given and $u(0) = u(L) = 0$.

What restrictions on α are necessary for the solution to exist ?

[6 Marks]

c) Show that the solution to part b) can be considered as the convolution of a function $G(x,x')$ with $f(x)$, and find the function $G(x,x')$ as a sum of trigonometric terms.

[11 Marks]

d) Show that $G(x,x_0)$ is the solution to the problem in part b) for the case where $f(x)$ is a delta-function at the point $x = x_0$.

[4 Marks]

3) a) Consider the one-dimensional problem

$$a(x)\, u''(x) + b(x)\, u'(x) + c(x)\, u(x) = f\,(x)$$

where $a(x)$, $b(x)$, $c(x)$ and $f(x)$ are known functions and are continuous in an interval $[\alpha,\beta]$.

Show that if the value of u and its first derivative are given at a single point $x_0 \in (\alpha,\beta)$ then the solution can be found at all other points in $[\alpha,\beta]$.

[10 Marks]

b) Consider the two-dimensional problem

$$a(x)\, u_{xx}(x) + 2\, b(x)\, u_{xy}(x) + c(x)\, u_{yy}(x) = f\,(x)$$

where $a(x)$, $b(x)$, $c(x)$ and $f(x)$ are known continuous functions.

The function $u(s)$ is given at points on a curve $\mathbf{t}(s)$, defined parametrically, together with its derivatives along $\mathbf{t}(s)$, and along the curve normal $\mathbf{n}(s)$. Show how to state conditions relating $a(x)$, $b(x)$ and $c(x)$ that determine whether or not the solution $u(x)$ can be found in the region neighbouring the curve $\mathbf{t}(s)$. Show that in 2D these conditions can be used to classify Partial Differential Equations, and give examples of each classification type.

[15 Marks]

TURN OVER

4) a) Describe the components of a *snake* model for image segmentation. Include in your description a definition of the *internal* and *image* forces and the method for optimisation of the model.

[15 Marks]

b) Suppose a 1024×1024 image has pixel values given by the function $I(x,y) = ((x-512)^2 + (y-512)^2)\exp(-(x-512)^2/400 - (y-512)^2/900)$. Discuss what contour will be found by a snake model acting on this image. Include in the discussion the initial state of the snake, and which energy terms will have the most significant effect. Discuss whether the solution is unique.

[10 Marks]

[**END OF PAPER**]