

4.

A communications experiment gave the following results:

Round-trip time msec	Processor speed MHz	Effective line speed
22	15	57.0
33	12	58.0
26	20	58.4
40	10	58.7
18	25	59.0
35	17	59.2
30	22	60.0
24	27	60.5
20	30	60.7

- a) Explain the principle of least square estimation. [5 marks]
- b) Give a detailed explanation of how the least square principles can be used to determine correlation between the variables in the survey. [10 marks]
- c) Determine the relationship between processor speed and round-trip time using the data from the mini-survey. [10 marks]

5.

- a) Describe the relationship between Markov chains and birth-death chains, and how they differ from other stochastic processes. [4 marks]
- b) What is meant by a steady state, and what are the conditions for a Markov chain reaching such a state? [3 marks]
- c) What is expressed by the Chapman-Kolmogorov equation? [4 marks]
- d) Derive an expression for the probability of having k births during the interval t for a pure birth chain with constant birth rate λ . [8 marks]
- e) Derive an expression showing how the intervals between births are distributed for a pure birth chain with constant birth rate. [6 marks]

6.

The maintenance division of a van rental company consists of three sections:

1. Valet service,
2. Mechanical repair workshop
3. Body repair workshop.

Cars returned by customers are evaluated by a works office where it is decided what action to take. Each van waits on average 20 minutes in the works office while the decision is made. Of the vans passing the works office

25% are released to customers. On average, a customer will use the van for 7 hours before returning it.

50% are directed to the valet service. The average treatment takes 40 minutes. After completion, the vans are returned to the works office.

22% have mechanical faults and are directed to the workshop. The repairs take on average 2 hours. After completion, the vans are returned to the works office.

3% have damaged bodywork and are directed to the body repair workshop. Repairs take on average 16 hours. After completion, the vans are forwarded directly to the valet service.

- a) Draw and label a diagram of the maintenance division. [5 marks]
- b) For each van, what are the respective visit ratios for the works office and each of the three sections? [5 marks]
- c) What is the total service demand for each division? [5 marks]
- d) Where is the bottleneck? [5 marks]
- e) If the utilisation of the valet service is 80%, what is the average response time if there are 100 customers? [5 marks]