Answer THREE questions
Calculators are permitted

1.

a) What characterises a static Markov chain?

[3 marks]

b) How do Markov chains differ from other stochastic processes?

[3 marks]

c) What is meant by a steady state, and what are the conditions for a Markov chain reaching such a state?

[3 marks]

d) What is expressed by the Chapman-Kolmogorov equation?

[3 marks]

e) Describe the relationship between Markov chains and birth-death chains.

[3 marks]

f) Derive an expression for the probability of having k births during the interval t for a pure birth chain with constant birth rate λ.

[10 marks]

g) Derive an expression showing how the intervals between births are distributed for a pure birth chain with constant birth rate.

[8 marks]

2.

Use the Cost Connection Matrix below and the heuristic proposed by Kruskal to construct an access network between 13 terminals (nodes 2-14) and a central server (node 1). The available lines are identical with a capacity of 22 traffic units per hour.

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	---	154	102	128	33	77	95	125	207	134	52	173	93	152
2	154	---	160	287	179	225	230	220	226	46	194	30	263	272
3	102	160	---	157	91	155	102	62	105	117	147	161	160	115
4	128	287	157	---	96	77	55	119	230	248	111	292	59	90
5	33	179	91	96	---	64	59	88	189	153	57	193	74	122
6	77	225	155	77	64	---	83	146	251	211	38	250	21	144
7	95	230	102	55	59	83	---	69	177	197	100	239	75	65
8	125	220	62	119	88	146	69	---	112	178	154	222	143	54
9	207	226	105	230	189	251	177	112	---	180	250	214	252	152
10	134	46	117	248	153	211	197	178	180	---	183	45	226	229
11	52	194	147	111	57	38	100	154	250	183	---	219	58	165
12	173	30	161	292	193	250	239	222	214	45	219	---	266	274
13	93	263	160	59	74	21	75	143	252	226	58	266	---	134
14	152	272	115	90	122	114	65	54	152	229	165	274	134	---

The mean traffic load from each node is as follows

2	3	4	5	6	7	8	9	10	11	12	13	14
2	3	10	2	10	3	4	5	4	10	7	7	7

What is the cost of this configuration?

[7 marks]

With how much would the cost differ if Prim's heuristic had been used?

[7 marks]

A rival company charges 25% more for their lines, but the line capacity is 75 units per hour. Is it possible to lower the cost by changing supplier?

[7 marks]

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	---	154	102	128	33	77	95	125	207	134	52	173	93	152
2	154	---	160	287	179	225	230	220	226	46	194	30	263	272
3	102	160	---	157	91	155	102	62	105	117	147	161	160	115
4	128	287	157	---	96	77	55	119	230	248	111	292	59	90
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14	152	272	115	90	122	114	65	54	152	229	165	274	134	---

It has been proposed to replace the access network with a backbone network. Assuming that the line capacities are those indicated in the table above, determine the maximum flow between node 9 and node 11.

[12 marks]

3.

Consider a network with M nodes connected by N links. Each node is assumed to be an independent server with Poisson distributed service rate. The capacity of the i^{th} link is given as C_i. The total traffic γ in the network as well as the way it is routed is assumed to be known.

a) Derive an expression for the mean delay across the network.

[11 marks]

b) Outline the approach proposed by Kleinrock to determine the capacities C_i that will minimise the total cost of the network provided the mean message delay for the network is constrained.

[16 marks]

c) What would the cost of this network solution be?

[7 marks]

4.

	A	B	C	D	E	F	G
A	--	1	9	7	∞	∞	∞
B	1	--	∞	4	6	∞	10
C	9	∞	--	3	∞	2	∞
D	7	4	3	--	∞	7	8
E	∞	6	∞	∞	--	∞	6
F	∞	∞	2	7	∞	--	2
G	∞	10	∞	8	6	2	--

The table above is the delay matrix for a network with 7 nodes. Infinity (∞) means there is no link connecting the two nodes.

a) Give a detailed account of the steps in the shortest path algorithm proposed by Dijkstra.

[8 marks]

b)

Find the geodesic between node A and each of the other nodes in the network graph above using Dijkstra's algorithm.

[8 marks]

c) Give a detailed account of the steps in the shortest path algorithm proposed by Ford and Fulkerson.

[9 marks]

d) The US Internet converted their routing scheme from using Ford-Fulkerson to Dijkstra's approach. What advantages and disadvantages would you expect from such a change?

[8 marks]

5.

The Alien Registration Office at Lunar House operates a ticket based waiting scheme. Upon arrival, each case (as people are called) picks a number from a central dispenser and waits to see one of the fifteen officers on duty. The time taken to deal with each case is assumed to be Exponentially distributed with an average of twelve minutes per case. The mean arrival rate is 40 cases per hour, assumed constant throughout the day.

What is the probability of a case finding all fifteen officers busy?

[8 marks]

How long time can a case expect to wait before being seen by an officer?

[12 marks]

Eighty percent of the cases can get their passports stamped immediately. To do this they must go the cashiers until where the passport is stamped after they have paid £5. The time taken to do this is exponentially distributed with an average of 6 minutes.
How many people will on average be queueing to have their passports stamped?

[8 marks]

2.5% of the cases are transferred to the supervisor. The time the supervisor spends on a case is exponentially distributed with an average of one hour per case. The waiting room holds three cases. If the waiting room is full, cases will be transferred to other facilities.
What is the probability of finding 2 cases waiting?

[5 marks]