UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Mathematical Studies

Level HE2 Examination

Module MS220 THEORY OF FINANCE

Time allowed - 2 hrs

Spring Semester 2007

Attempt THREE questions
If a candidate attempts more than THREE questions only the best THREE questions will be taken into account.

In all your answers, give a short justification or method of calculation that leads to your answer. Numerical answers should be rounded to the nearest penny.

Formula sheet:

Level annuities at constant interest rate i:

$$\begin{array}{ll} \text{PV in arrears} & a_{\overline{n}|} = \frac{1-v^n}{i}. \\ \text{PV in advance} & \ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|} \\ \text{FV in arrears} & s_{\overline{n}|} = \frac{(1+i)^n-1}{i} \\ \text{FV in advance} & \ddot{s}_{\overline{n}|} = (1+i)s_{\overline{n}|}. \end{array}$$

p**-thly payable** level annuities:

$$\begin{array}{ll} \text{PV in arrears} & a^{(p)}_{\overline{n}|} = \frac{i}{i^{(p)}}a_{\overline{n}|}. \\ \text{PV in advance} & \ddot{a}^{(p)}_{\overline{n}|} = \frac{d}{d^{(p)}}\ddot{a}_{\overline{n}|}. \\ \text{FV in arrears} & s^{(p)}_{\overline{n}|} = \frac{i}{i^{(p)}}s_{\overline{n}|}. \\ \text{FV in advance} & \ddot{s}^{(p)}_{\overline{n}|} = \frac{d}{d^{(p)}}\ddot{s}_{\overline{n}|}. \end{array}$$

Bond Price Formula

$$P = \frac{F}{(1+\frac{\lambda}{m})^n} + \frac{C}{\lambda}\left(1 - \frac{1}{(1+\frac{\lambda}{m})^n}\right).$$

Macaulay's Duration Formula

$$D = \frac{1+y}{my} - \frac{1+y+n(c-y)}{mc[(1+y)^n-1]+my}.$$

Probability distribution for the normal distribution $\mathcal{N}(\mu, \sigma^2)$ is

$$\rho_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}.$$

Table for the standard normal distribution: $\int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx$.

z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Question 1

Jim wants to build up a pension fund for his retirement, 20 years from now. At that time, he would like to have an amount of £110,000. A pension company allows him to build up this fund by annuities at interest rate 6%.

(a) If Jim pays into his fund annually in advance, how much is his annual instalment? [3]

(b) Explain what $s_{\overline{n}|}$ and $\ddot{s}_{\overline{n}|}$ are. Explain why $s_{\overline{n+1}|} = \ddot{s}_{\overline{n}|} + 1$. [4]

(c) Suppose Jim agrees to pay his instalments monthly in advance. How much would each instalment be? [4]

(d) What would Jim's monthly instalment be if the interest was converted quarterly? [5]

(e) Jim agrees with the company that at the moment of his retirement, he is paid a lump sum of £30,000. The rest of his fund will be paid to him by annual annuities, over a period of 15 years, starting at the end of the 4th year after his retirement. The interest rate is still 6%. For how much will his annual pension cheque be? [5]

(f) Suppose that Jim died just after receiving his 5th pension cheque, and that the remainder of the pension fund is paid to Jim's next of kin. (That is: the pension company pays the outstanding capital.) How much would this payment be? [4]

Question 2

On 1st January 2008, a farmer decides to invest £23,500 in a shed and equipment for growing mushrooms. He expects to sell the harvest for £8,000 on December 31 of each of the years 2008, 2009 and 2010, and at end of the project (31 December 2010), he sells his equipment for £2,000.

(a) Make a table of the annual net cashflows and the total cashflow, and compute the present value if the interest rate is 6% per annum. Based on your answer, what is your advice to the farmer? [5]

(b) Compute the pay-back time. [3]

(c) What is the annualised cost of the project and is the project viable from this point of view? [4]

(d) Write down the Equation of Value. Sketch the PV as function of v. Indicate the exact values of the present values for $v = 0$ and $v = 1$.

Explain the effect on the IRR if

(i) the farmer cannot sell his equipment at the end of the project,

(ii) the interest rate goes up from 6% to 7%. [6]

(e) It is more realistic to assume that the income from the mushrooms is paid continuously over the year, instead of at the end of the year. Describe the payment model that arises from this change, and write down the resulting Equation of Value. [3]

(f) Show that the IRR in part (e) is very close to 6.57%. [4]

Question 3

Kathy puts £12,000 in a personal pension fund, which she hopes to cash in after 30 years.

(a) Assume that the interest rate decreases over time expressed by a force of interest $\delta(t) = (8 - 0.1t)\%$. What is the payout? [4]

(b) In year 20, what annual interest rate would be equivalent to the force of interest in part (a)? [4]

(c) Instead of a known interest rate, the interest rates i_k in years $k = 1, \ldots, 30$ are independent random variables. Assuming that the i_k are uniformly distributed on the interval $[5\%, 9\%]$, what is the expected payout after 30 years? [3]

(d) Show that the expectation of $\log(1 + i_k)$ is approximately $\mu = 0.0676$. [4]

(e) What is the approximate distribution of the pay-out? Justify your answer, and also indicate the parameters. (You may assume that the variance of $\ln(1 + i_k)$ is approximately $\sigma^2 = 0.0001165$.) [4]

(f) Kathy has to choose between this pension fund as in (c) and a savings account with a fixed interest rate of 7%. What is the probability that the pension fund pays out better than the savings account? [6]

Question 4

(a) A certain 4-coupon bond A has face value £100 and is sold for £92. The maturity is 15 years, and the yield to maturity is found to be 9%. Show that the coupon rate is approximately 8%. [4]

(b) After 10 years, you want to sell the bond, and the yield you require is $\lambda = 8\%$. At what price should you sell?

The theory states that if a bond is sold at par, then the yield is the same as the coupon rate. (Here: both 8%). Explain why in this case, the selling price is not £100. [4]

(c) Compute the current yield (CY), and explain the effect on the CY of the early sale of the bond. [4]

(d) Use Macaulay's formula to show that the duration of bond A at a yield of $\lambda = 9\%$ is approximately 8.58 years. Explain how the duration is affected if the yield increases. [5]

(e) Let B be a bond with duration of 15 years, sold for £80. Let X_A and X_B be the total investments in bonds A and B respectively. Explain how the duration of the portfolio of these bonds is computed.

What are the possible values that the duration of a portfolio of bonds A and B can take? [4]

(f) Suppose you need the portfolio as an investment to pay £80,000 in 10 years' time. If interest rates and yields are 9% throughout, how many shares of A and B do you need to immunise your portfolio against this future obligation? [4]

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