UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Mathematical Studies

Level HE2 Examination

Module MS220 THEORY OF FINANCE

Time allowed - 2 hrs Spring Semester 2006

Attempt THREE questions
If a candidate attempts more than THREE questions only the best THREE questions will be taken into account.

In all your answers, give short justification or method of calculation that leads to your answer. Numerical answers should be rounded to the nearest penny.

Formula sheet:

Level annuities at constant interest rate i:

$$\begin{array}{ll} \text{PV in arrears} & a_{\overline{n}|} = \frac{1-v^n}{i}. \\ \text{PV in advance} & \ddot{a}_{\overline{n}|} = (1+i)a_{\overline{n}|} \\ \text{FV in arrears} & s_{\overline{n}|} = \frac{(1+i)^n-1}{i} \\ \text{FV in advance} & \ddot{s}_{\overline{n}|} = (1+i)s_{\overline{n}|}. \end{array}$$

p**-thly payable** level annuities:

$$\begin{array}{ll} \text{PV in arrears} & a^{(p)}_{\overline{n}|} = \frac{i}{i^{(p)}}a_{\overline{n}|}. \\ \text{PV in advance} & \ddot{a}^{(p)}_{\overline{n}|} = \frac{d}{d^{(p)}}\ddot{a}_{\overline{n}|}. \\ \text{FV in arrears} & s^{(p)}_{\overline{n}|} = \frac{i}{i^{(p)}}s_{\overline{n}|}. \\ \text{FV in advance} & \ddot{s}^{(p)}_{\overline{n}|} = \frac{d}{d^{(p)}}\ddot{s}_{\overline{n}|}. \end{array}$$

Bond Price Formula

$$P = \frac{F}{(1+\frac{\lambda}{m})^n} + \frac{C}{\lambda}\left(1 - \frac{1}{(1+\frac{\lambda}{m})^n}\right).$$

Macaulay's Duration Formula

$$D = \frac{1+y}{my} - \frac{1+y+n(c-y)}{mc[(1+y)^n-1]+my}.$$

Probability distribution for the normal distribution $\mathcal{N}(\mu, \sigma^2)$ is

$$\rho_{\mu,\sigma}(x) = \frac{1}{\sqrt{2\pi}\sigma}e^{-(x-\mu)^2/2\sigma^2}.$$

Table for the standard normal distribution: $\int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}}e^{-x^2/2}\,dx$.

z	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	.5000	.5398	.5793	.6179	.6554	.6915	.7257	.7580	.7881	.8159
1	.8413	.8643	.8849	.9032	.9192	.9332	.9452	.9554	.9641	.9713
2	.9772	.9821	.9861	.9893	.9918	.9938	.9953	.9965	.9974	.9981
3	.9987	.9990	.9993	.9995	.9997	.9998	.9998	.9999	.9999	1.0000

Question 1

Anne buys a mortgage for £180, 000 to be paid back over 20 years. The annual interest rate is 8% throughout.

(a) How much will her instalment in year 7 be if she pays by annuities, annually in arrears? [3]

(b) Suppose that inflation is 3% throughout the 20 years. What is the present purchase power of Anne's mortgage payments, i.e. the PV, if the effect of inflation is included?

(N.B. inflation plays a role only in this part; not in the other parts of Question 1.) [4]

(c) How much will her instalment in year 7 be if she pays by interest on outstanding capital and repays the capital in equal portions? [3]

(d) Explain in words what $d^{(p)}$ means. Calculate $d^{(12)}$ for $i = 8\%$. [4]

(e) Anne can afford to pay £1, 500 every month, starting two years after the purchase of the mortgage. Assuming she pays by annuities monthly in advance, for how many years will she be paying? [5]

(f) Suppose that Anne pays by annual annuity instalments as in part (a). After paying 12 instalments, she gets the chance to refinance her mortgage at a new interest rate of 6%. The penalty for breaking the contract with her first bank (to which she has to repay the outstanding capital) is £4, 500. What is the outstanding capital after 12 years and would you advise Anne to refinance? [6]

Question 2

(a) Norman can invest £100 in stock S with annual yield i. The expected value of the yield is 5%. Let F be the future value of the stock in three years' time? What is the expected value of F? [2]

(b) Assume that the odds of i are 4%, 5% and 6%, each with probability $\frac{1}{3}$, and that, once i is determined, it remains fixed for 3 years. Show that the variance of the value of £100 of S after three years is approximately 21.9£^{2}. [4]

(c) If instead of part (b), the distribution of i was uniform on $[4\%, 6\%]$, would Norman's project be riskier or not? Explain your answer. [4]

(d) If $\log(1 + i)$ has distribution $\mathcal{N}(0.049, 0.02)$, what distribution does T have, and what are the parameters? [4]

(e) Norman invests £50,000 in 500 shares. The shares X_k, $k = 1, \dots 500$, all have principal value £100 and annual yields i_k, where the i_k are independent random variables having the same distribution. Consequently, the future values T_k of the shares X_k in three years' time are random variables. Write $\mu = \mathbb{E}(T_k)$ and $\sigma^2 = \mathrm{Var}(T_k)$. Name and state the theorem predicting the distribution of the value of Norman's portfolio in three years' time. [5]

(f) Given that $\sigma^2 = 20$£^{2}, specify the parameters of the distribution of Norman's portfolio. How much can $\mathbb{E}(X_k)$ be at most, to make it 95% sure that Norman's portfolio will be worth no more than £55,000 in three years' time? [6]

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Question 3

On 1st January 2006, a car rental company buys 12 cars for £20,000 each. Every year, the income from renting out this fleet is £90,000, which is booked at the end of the year. At the end of the second year, 6 of the cars are sold second-hand for £5,000 each and 6 new cars are bought, again for £20,000 each. At the end of year 3, the whole fleet of cars is sold for £90,000, and the project is closed.

(a) Make a table of the annual net cashflows and the total cashflow. Compute the ROI and annual ROI based on the net cashflows. [4]

(b) Assume that the interest rate is 5% throughout. Compute the present value and the future value (on December 31 2008) of the project. [4]

(c) Compute the annualised cost of the project based on the net cashflows and 5% interest. Is the project viable from this viewpoint? Explain your answer. [4]

(d) Write down the Equation of Value. Sketch the PV as function of v. Indicate the exact values of the present values for $v = 0$ and $v = 1$.

Explain why there is a unique IRR. [5]

(e) The income from renting out the cars is obtained throughout the year, so it is unrealistic to book these cashflows at the end of each year. Instead, a mixed discrete-continuous time model is more appropriate. Describe such a model, and compute the rate of payment ρ for the cash inflow. [4]

(f) Write down the Equation of Value for the model obtained in part (e). Explain in words what the effect on the IRR is (compared with the IRR in the discrete time model) and why. [4]

Question 4

A certain 4-coupon bond A has face value £100. Its Price Yield Curve is shown in Figure 1.

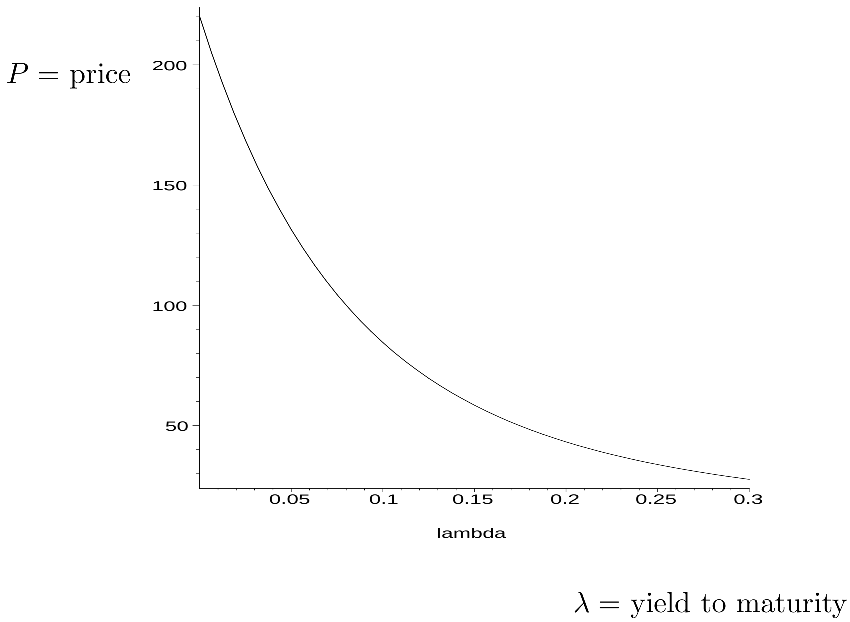

Figure 1: Price Yield Curve for a 4-coupon bond.

(a) Explain from the graph why the coupon rate is approximately 8%.
Estimate the bond's maturity in years. Explain your answer. [4]

(b) Suppose you bought the bond for £90. After 10 years you want to sell the bond, and the yield you require is 9%. At what price should you sell? [4]

(c) Derive the Price Sensitivity Formula. [5]

(d) Use Macaulay's formula to show that the duration of bond A at a yield of $\lambda = 9\%$ is approximately 8.58 years. Explain how the duration is affected if the yield increases. [5]

(e) Bond B is a zero-coupon bond with face value of £80 and maturity of 15 years. Compute the duration of B and its price if the yield $\lambda = 9\%$. [3]

(f) Suppose you want to use bonds A and B to create a bond portfolio. If you need your portfolio to have a duration of 12 years, and you are investing £25,000, how many shares of each bond should you buy? [4]

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