UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Mathematical Studies

Level HE1 Examination

Module MS106 CLASSICAL DYNAMICS

Time allowed – 2 hrs Spring Semester 2008

Attempt THREE questions
If a candidate attempts more than THREE questions only the best THREE questions will be taken into account.

Question 1

(a) In an athletics competition a javelin is thrown with a speed of u at an angle of α to the horizontal.

Ignore air resistance and consider the javelin as a point mass of mass m.

(i) Show that the javelin lands at a distance

$$\frac{2u^2 \sin\alpha\cos\alpha}{g},$$

measured horizontally from its starting point. [8]

(ii) Hence or otherwise deduce the angle α that the javelin thrower should throw the javelin at in order to maximise her chances of winning a medal. [3]

(b) A ball of mass m is thrown vertically upwards with speed V. Model air resistance by a resistive force that is proportional to the speed of the ball with constant of proportionality k.

(i) Draw a diagram showing all the forces acting on the ball. [2]

(ii) Show that the velocity of the ball is given by

$$\mathbf{v} = \left\{-\frac{mg}{k} + \left(\frac{mg}{k} + V\right) \mathrm{e}^{-\frac{kt}{m}}\right\} \mathbf{j},$$

where $\mathbf{j}$ is a unit vector pointing upwards. [6]

(iii) Show that the greatest height reached is

$$\frac{mV}{k} - \frac{m^2 g}{k^2} \ln\left(1 + \frac{kV}{mg}\right).$$

[6]

Question 2

(a) A sledge is at rest at the top of a hill that is inclined at an angle of 30° to the horizontal. Assume that the frictional force betwen the sledge and the ground is negligible.

(i) Draw a diagram indicating clearly any forces acting on the sledge. [2]

(ii) It is 100 metres to the bottom of the hill. How long does it take for the sledge to reach the bottom of the hill? (Take $g = 10$ m/s^2.) [5]

(b) A train of mass m enters some sidings at a speed U. As it enters the sidings the brakes are applied and the train experiences a braking force of mk. In addition, there is a frictional force between the track and the train with coefficient of friction μ.

(i) Draw a diagram indicating all the forces acting on the train. [3]

(ii) Find an expression for the time it takes for the train to stop. [4]

(iii) Show that the train travels a distance of

$$\frac{1}{2}\left(\frac{U^2}{k + \mu g}\right)$$

in this time. [3]

(iv) Calculate the work done by the braking force. [2]

(v) Calculate the work done by the frictional force. [2]

(vi) Calculate the change in kinetic energy of the train. [2]

(vii) Comment on how your answers to (iv), (v) and (vi) relate to each other. [2]

Question 3

(a) A car travelling at 10 m/s bumps into a second, stationary, car of the same mass. If the coefficient of restitution between the two cars is 1/2, calculate the speeds of the two cars after the collision. You may assume that the collision is one-dimensional. [8]

(b) A mass m_A collides with a mass m_B in an inelastic collision. Let the initial velocity of A be $\mathbf{u_A}$ and the initial velocity of B be $\mathbf{u_B}$.

(i) If the final velocity of the combined mass is $\mathbf{v}$ show that

$$m_A \mathbf{u_A} + m_B \mathbf{u_B} = (m_A + m_B)\mathbf{v}$$

[2]

(ii) If $\mathbf{u_A}$ is perpendicular to $\mathbf{u_B}$ deduce that

$$u_A = \left(\frac{m_A + m_B}{m_A}\right) v \cos\theta,$$

where θ is the angle between $\mathbf{v}$ and $\mathbf{u_A}$, u_A is the magnitude of $\mathbf{u_A}$ and v is the magnitude of $\mathbf{v}$. [3]

(iii) Derive a similar expression for u_B, where u_B is the magnitude of $\mathbf{u_B}$. [2]

(iv) Write down expressions for the total initial and final kinetic energy of the masses. [5]

(v) Show that if $m_A = m_B$ then the final kinetic energy is half the initial kinetic energy. [5]

Question 4

(a) Tom and his older sister, Rebecca, enter a playground. Tom is of mass 20 kg and sits on the end of a seesaw of total length 4m. Where should his older sister of mass 30 kg sit in order to balance the seesaw? [4]

(b) A roundabout in the playground consists of a horizontal disc that pivots about the centre of the disc. A bag of mass m is placed on the roundabout at a distance of a from the centre. The coefficient of static friction between the roundabout and the bag is μ_s.

Tom starts to spin the roundabout. What is the maximum angular speed that he can spin the roundabout without the bag starting to slide? [5]

(c) The Star Ship Enterprise is spiralling in towards a strange planet with a trajectory given by

$$r = A\exp(-k\phi), \qquad \dot{\phi} = B\exp(2k\phi). \tag{1}$$

(i) Show that the acceleration of the Enterprise in polar coordinates is

$$-\frac{A^4B^2(1+k^2)}{r^3}\hat{\mathbf{r}}.$$

[8]

(ii) If the Enterprise is of mass M, what is the force that acts on the Enterprise? [2]

(iii) From equations (1) and given that $\phi = 0$ when $t = 0$, find $\phi(t)$ and hence $r(t)$. Hence find when the Enterprise will hit the surface of the planet. The planet has radius R. [6]

Question 5

(a) (i) State what is meant by a central force. [2]

(ii) Show that for a central force angular momentum is conserved. [3]

(iii) Explain why for motion of a planet around a fixed mass, as modelled using Newton's law of gravity, it is sufficient to consider motion in two rather than three dimensions. [2]

(b) The total mechanical energy E for a meteor of mass m orbiting a large fixed mass M is given by

$$E = -\frac{GMm}{r} + \frac{m}{2}\left(\dot{r}^2 + r^2\dot{\phi}^2\right).$$

The orbit of the meteor is given by

$$\frac{1}{r} = \frac{1}{l} + A\cos\phi,$$

where $l = L^2/GMm^2$, L is the magnitude of the angular momentum and G is the universal gravitation constant.

(i) Show that

$$L = mr^2\dot{\phi},$$

and that

$$\dot{r} = \frac{AL\sin\phi}{m}.$$

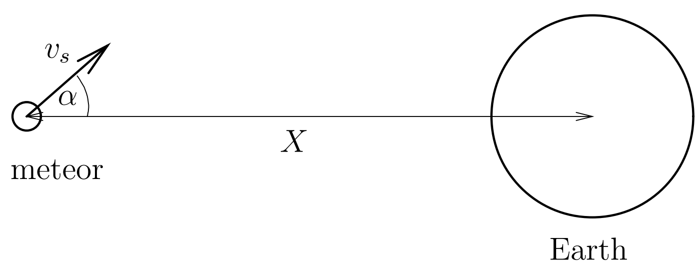
[4]

(ii) Hence show that

$$E = \frac{1}{2}\frac{L^2A^2}{m} - \frac{1}{2}\frac{G^2M^2m^3}{L^2}.$$

[8]

(iii) A meteor is spotted approaching Earth. When first seen it is at a distance of X and travelling at a speed of v_s as shown below.



Assuming that the mass of the Earth and the gravitational constant G are known, outline how you would deduce the closest approach of the meteor to the Earth. [6]

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