UNIVERSITY OF SURREY©

B. Sc. Undergraduate Programmes in Mathematical Studies

Level HE1 Examination

Module MS106 CLASSICAL DYNAMICS

Time allowed – 2 hrs

Spring Semester 2007

Attempt THREE questions
If a candidate attempts more than THREE questions only the best THREE questions will be taken into account.

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Question 1

A ball is thrown from a height h at a wall a horizontal distance d away. The ball is thrown with a speed v and at an angle α measured upwards from the horizontal.

(a) Neglecting air resistance,

(i) show that the ball hits the wall at a height H on the wall where

$$H = d \tan\alpha - \frac{1}{2}\frac{gd^2}{v^2}\sec^2\alpha + h.$$

[8]

(ii) Show that in order for the ball to hit the wall as high as possible, the ball must be thrown at an angle $\tan^{-1} v^2/gd$. [7]

(iii) Hence find an expression for the maximum height that the ball can hit the wall. [4]

(b) Now suppose that air resistance proportional to the velocity with constant of proportionality k is included.

(i) write down, but do not solve, the equations of motion for the horizontal and vertical directions. [4]

(ii) How would you expect air resistance to affect the answer in (a) for the maximum height reached up the wall? [2]

Question 2

(a) A block slides down a slope of angle α with an initial speed V down the slope.

(i) Show that the acceleration of the block down the slope is given by

$$g\left(\sin\alpha - \mu\cos\alpha\right),$$

where μ is the coefficient of friction. [5]

(ii) Hence show, assuming that the slope is sufficiently long, the block will eventually stop sliding if $\mu > \tan\alpha$. [2]

(iii) Assuming $\mu > \tan\alpha$, find how long the block takes to come to a stop and how far it goes in this time. [8]

(b) In a one-dimensional perfectly elastic collision a sphere of mass m travelling at a velocity of $2u$ hits a second sphere of mass $2m$ travelling at velocity u. Find the velocities of each of the spheres after the collision in terms of u. [10]

Question 3

(a) A ball is dropped from a height h. Assuming that air resistance is negligible

(i) calculate the velocity of the ball when it hits the floor. [3]

(ii) If the coefficient of restitution for collision between the floor and the ball is e, calculate the fraction of the total initial mechanical energy that is lost in each collision. [3]

(iii) Describe the subsequent motion of the ball in each of the three cases $e = 1$, $e = 0$ and $0 < e < 1$. [3]

(b) Consider the force

$$\mathbf{F} = -kx\mathbf{i} - ky\mathbf{j}.$$

(i) Show that it is a conservative force by finding a suitable potential function $V(x, y)$. [4]

(ii) The force $\mathbf{F}$ acts on a point mass m. Write down the equation of motion for the mass m. [2]

(iii) If at $t = 0$ the mass starts from rest at $x = 1, y = 1$ find the subsequent position of the mass as a function of time. [5]

(iv) Describe the trajectory of the mass. [2]

(v) Calculate the total mechanical energy of the mass. Explain why your answer does not depend on time. [3]

Question 4

(a) A ladder of mass 20 kg and length 6 metres leans against a rough wall at an angle of inclination $\theta = \tan^{-1} 2$ to the horizontal.

A man of mass $80kg$ wishes to climb 4.5 metres up the ladder. Suppose the coefficient of friction between the wall and the ladder and between the floor and the ladder is μ.

(i) Draw a diagram showing all the forces acting on the ladder. [2]

(ii) Show that, at the point of slipping, the frictional force between the wall and the ladder is given by

$$\frac{100\mu^2 g}{1 + \mu^2},$$

where g is the acceleration due to gravity. [6]

(iii) Find the minimum value of μ required for the ladder not to slip. [6]

(b) (i) A particle of mass m slides up the inside of a smooth hollow sphere of internal radius a and centre O. At the lowest point, the particle has a speed of $\sqrt{\frac{7}{2}ag}$.

(i) Draw a diagram showing all the forces acting on the particle. [2]

(ii) Find the value of the angle which OP makes with the upward vertical at the point when the particle leaves the sphere. [9]

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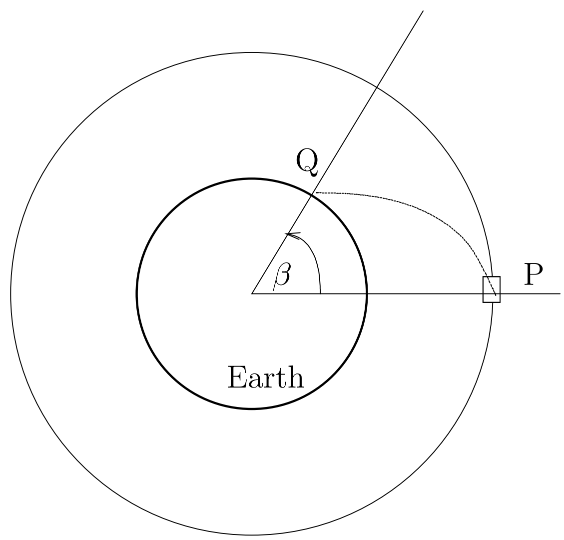
Question 5

A space shuttle is orbiting the Earth in a circular orbit of radius R metres. When it reaches point P in the diagram shown, it fires it's braking rockets. This has the effect of reducing the speed by Δv m/s. As a result, the space shuttle follows a new trajectory, as shown, hitting the Earth at Q.



(a) Show that the angular momentum of the space shuttle in it's original circular orbit is given by

$$\sqrt{GMm^2R},$$

where M is the mass of the Earth, m is the mass of the space shuttle. [4]

(b) Hence or otherwise find the speed of the space shuttle at P in its original orbit and deduce that the new speed is given by

$$\sqrt{\frac{GM}{R}} - \Delta v.$$

[6]

(c) Given that $R = 6.90 \times 10^6$m and $\Delta v = 1000$m/s, find the value of β at which the space shuttle hits the Earth.

(You will also need that $M = 5.98 \times 10^{24} kg$, $G = 6.67 \times 10^{-11} Nm^2/kg^2$ and that the radius of the Earth is $6.37 \times 10^6 m$). [15]

(You may use the fact that a mass m orbiting a mass M, follows a trajectory given by

$$r = \frac{l}{1 + lA\cos\phi},$$

where $l = L^2/GMm^2$, where L is the angular momentum.)

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