MAT2013 Unassessed Exercise 1– Semester 1 2011/12

Module:	MS Mathematical Statistics
Lecturer:	K Young
Due date:	Tuesday 1st November (in lecture) 2011

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1. Let N be a random variable with geometric (π) distribution.
   (a) Show that $P(N > k) = (1-\pi)^k$.
   (b) Hence, find an expression for $P(N > a+b|N > a)$, where a, b are natural numbers, and comment on your result.
   (c) A biased die has probability $\frac{2}{5}$ of scoring a six. Let N represent the number of throws up to and including the first six. Given that the first 4 throws do not result in a six, write down the probability that the total number of throws up to and including the first six will be greater than 7.

2. Let the continuous random variable V have the p.d.f.
$$\begin{aligned} f(v) &= 2v, \quad 0 < v < 1, \\ &= 0 \text{ elsewhere.} \end{aligned}$$
   Obtain the p.d.f. of the random variable $W = 8V^3$.

3. Let the continuous random variable X have the p.d.f.
$$\begin{aligned} f(x) &= 1, \quad 0 < x < 1, \\ &= 0 \text{ elsewhere.} \end{aligned}$$
   Obtain the p.d.f. of the random variable $Y = -2\ln X$.

4. Determine the constant c so that $f(x) = cx(3-x)^4,\ 0 < x < 3$, zero elsewhere, is a p.d.f.

5. Let X have the p.d.f.
$$\begin{aligned} f(x) &= \frac{x-1}{2}, \quad 1 < x < 3, \\ &= 0 \text{ elsewhere.} \end{aligned}$$
   Find a transformation $h(x)$ so that $Y = h(X)$ has a uniform distribution over the interval (0,1).

6. A quality characteristic X of a manufacured item is a continuous random variable with p.d.f.
$$\begin{aligned} f(x) &= 2\lambda^{-2}x, \quad 0 < x < \lambda, \\ &= 0 \text{ elsewhere.} \end{aligned}$$
   where $\lambda > 0$. Obtain the distribution function for X and hence find an expression for the median, m, in terms of λ. Find the mean and variance of X.

   [Hint: $F(m) = 0.5$]

7. The random variable $Y \sim \text{Po}(\lambda)$. Obtain $E[Y^3]$.

   [Hint: find $E[Y(Y-1)(Y-2)]$ and use $E[Y] = \text{Var}[Y] = \lambda$]