MAT2013 Unassessed Assignment 2– Semester 1 2011/12

Module:	MAT2013 Mathematical Statistics
Lecturer:	K Young
Due date:	Lecture on Tuesday 6th December 2011

Please show all working clearly

1. Determine the mean and the variance of X if the moment generating function of X is given by
   (a) $M_X(z) = \frac{1}{(1-3z)}$;
   (b) $M_X(z) = \frac{3}{(3-z)}$.

2. Use the Cauchy-Schwartz inequality to prove that
   (a) $E(X^{-2}) \geq \{E(X^2)\}^{-1}$, and
   (b) $E\{(X-Y)Y\} = 0 \Rightarrow E(Y^2) \leq E(X^2)$.

3. X is a random variable with mean 33 and variance 16. use Chebyshev's inequality to find
   (a) A lower bound for $P(23 < X < 43)$.
   (b) An upper bound for $P(|X - 33| \geq 14)$.

4. Let X have a geometric (π) distribution. Obtain the mgf of X.
   Given that the negative binomial (n, π) distribution has mgf
   $$M_n(z) = \frac{(\pi e^z)^n}{\{1-(1-\pi)e^z\}^n},$$
   deduce that the negative binomial random variable can be regarded as a sum of n iid random variables with a geometric (π) distribution.

5. Let $X_1, X_2, \ldots, X_n$ be independent random variables, such that for $i = 1, 2, \ldots, n$, X_i has a Gamma(α_i, β) distribution.
   Given that the moment generating function for X_i is $\left(\frac{\beta}{\beta - z}\right)^{\alpha_i}$, show that $Y = \sum_{i=1}^{n} X_i$ has a Gamma distribution and write the parameters in terms of α_i and β.

6. Determine the constant c in each of the following so that each $f(x)$ is a Beta random variable;
   (a) $f(x) = cx(1-x)^3, \quad 0 < x < 1,$ zero elsewhere;
   (b) $f(x) = cx^4(1-x)^5, \quad 0 < x < 1,$ zero elsewhere;
   (c) $f(x) = cx^2(1-x)^8, \quad 0 < x < 1,$ zero elsewhere;