

Assignment 1 MAT2004 Real Analysis II – 2011/12

Web pages: <http://www.maths.surrey.ac.uk/modules/MAT2004.html>
Lecturer: Gianne Derks
Due date: Wednesday 26 October 2011, start of lecture

- This assignment is for feedback only and is not part of the assessment of the module. You have to hand in solutions as part of your participation requirement.
- Hand in before the start of the lecture on **Wednesday 26 October 2011**.
- There are 4 questions, give full workings with your answers.
- It can be a good idea to discuss possible solution strategies with other students, but you are advised to write the solutions itself independently. However, if you work in a group and also write the solutions together, then it is sufficient to hand in just one piece of work with all names of the group members on it.

Question 1 Find $\lim_{x \rightarrow 3} \frac{3x+1}{x+2}$ and use the definition to show that this is the limit.

Question 2 Let $\lim_{x \rightarrow x_0} f(x) = l$ and $\lim_{x \rightarrow x_0} g(x) = m$. Show that $\lim_{x \rightarrow x_0} \max\{f(x), g(x)\} = \max\{l, m\}$.
Hint: show that $\max\{f(x), g(x)\} = \frac{1}{2}(f(x) + g(x) + |f(x) - g(x)|)$.

Question 3 Using the Intermediate Value Theorem, show that the equation $1 - \cos \pi x = x^2$ has a strictly positive solution.

Question 4 Define $f : (0, \infty) \rightarrow \mathbb{R}$ as $f(x) = x \ln x$. It can be shown that $f(x) \leq x\sqrt{x}$ for $x \geq 4$ (you don't have to do this).

(a) Show that $f(x) \geq -\sqrt{x}$ for $0 < x < \frac{1}{4}$.

(b) Show that $\lim_{x \downarrow 0} f(x)$ exists and find the limit.
