Faculty of Computing, Engineering & Technology

Session 2009/10
Semester 2

STAFFORDSHIRE UNIVERSITY

Module Code:	**CE62014-2**
Module Title:	**ENGINEERING MATHEMATICS WITH APPLICATIONS 2**
Date:	**TUESDAY 4TH MAY 2010**
Time:	**2:00 PM – 4:00 PM**
Duration:	**2 HOURS**
Examiner:	**DR.P.A. LEWIS/PROF B.L. BURROWS**
Extension:	**3549/3420**

INSTRUCTIONS TO CANDIDATES:

This paper consists of THREE questions. You must answer ALL questions.

The marks allocated for each question, or for its parts, are shown on the right.

All personal details must be completed at the top of every Answer Booklet. Please ensure that the corner of the booklet is folded down and sealed.

Every question attempted should be clearly marked in the space provided on the front of the Answer Booklet.

There is a formula sheet attached to the back of this paper

CANDIDATES WILL REQUIRE:

- Answer booklet
- Examination paper
- Mathematics Formula Booklet

1. (a) Show that the eigenvalues of the matrix

$$A = \begin{pmatrix} 3 & -1 & 0 \\ 4 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

are -2, -1 and 2. Hence determine a linearly independent set of eigenvectors for this matrix.

[10 marks]

(b) The components of the displacement of an object x, y and z satisfy the coupled first order differential equations

$$\begin{aligned} \frac{dx}{dt} &= 3x - y \\ \frac{dy}{dt} &= 4x - 2y \\ \frac{dz}{dt} &= -2z \end{aligned}$$

Using your results from (a) above, determine the General Solutions for the displacements. Describe the behaviour of the dominant term in the solutions for each component of the displacement and illustrate your explanation by considering the solutions for the following two sets of initial conditions:

$$(i) \quad x(0) = 1, \;\; y(0) = 1, \;\; z(0) = 1$$

and

$$(ii) \,. \;\; x(0) = 1, \;\; y(0) = 4, \;\; z(0) = 0$$

[10 marks]

2. Use integration by parts to show that

$$\int_{-1}^{1} x^2 \cos(n\pi x)dx = 2\int_{0}^{1} x^2 \cos(n\pi x)dx = \left(-\frac{4}{n\pi}\right)\int_{0}^{1} x \sin(n\pi x)dx$$

[5 marks]

and

$$\int_{0}^{1} x \sin(n\pi x)dx = \frac{(-1)^{n+1}}{\pi n}$$

[5 Marks]

Expand the function $f(x) = x^2 + 1$ of period 2, in a Fourier series, using the interval $-1 \leq x \leq 1$. (You may make use of the integrals evaluated above).

[6 marks]

Write down the series and the resulting series obtained when $x = 0$. From this second series show how we may deduce that

$$-\frac{\pi^2}{12} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

[4 marks]

3. The transient function $f(t)$ is defined by

$$f(t) = (u(t+1) - u(t-1))\cos(\pi t)$$

By making use of the result

$$2\cos(\pi t)\cos(\omega t) = \cos((\pi - \omega)t) + \cos((\pi + \omega)t)$$

or otherwise show that the Fourier transform of $f(t)$ is given by

$$F(\omega) = \frac{\sin(\pi - \omega)}{\pi - \omega} + \frac{\sin(\pi + \omega)}{\pi + \omega}$$

[6 marks]

Use this result and the properties of Fourier transforms to find

(a) FT { $f(t-3)$ }
(b) FT{ $f(3t+2)$ }
(c) FT{ $e^{3jt}f(t)$ }
(d) FT{ $tf(t)$ }

[8 marks]

Find also, using the symmetry property

$$FT\{F(t)\}$$

What is the range of ω for which this has non-zero values?

[3 marks]

Find the Fourier transforms of

(a) $e^{-t^2}\delta(t)$ and
(b) $(u(t+1) - u(t+0.5))\delta(t)$

[3 marks]