The principles of General Relativity

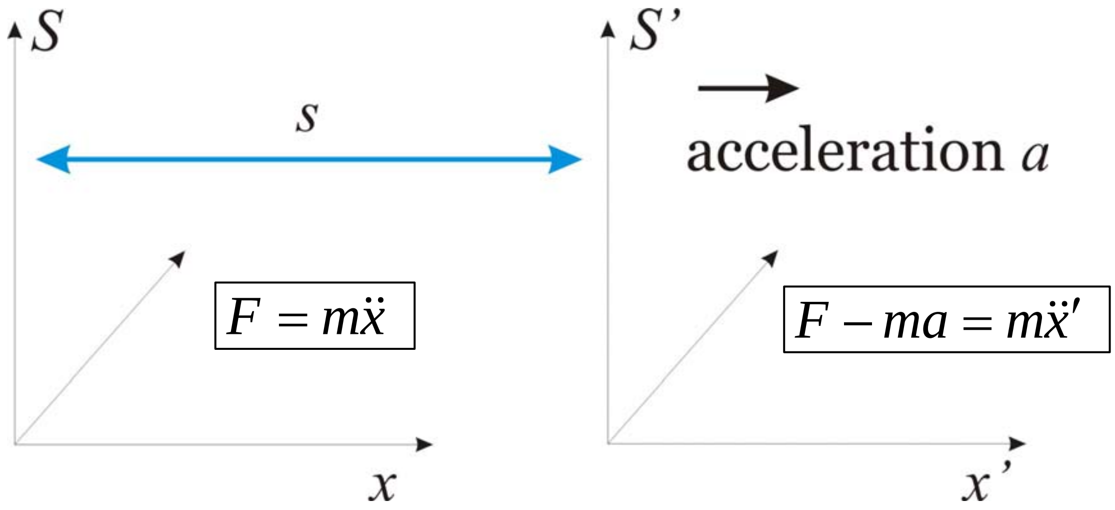
Principles of GR

Before we derive the field equations of general relativity, it is useful to take a closer look at the principles and ideas that inspired Einstein.

1. Mach principle
2. Equivalence principle
3. Covariance
4. Minimal gravitational coupling
5. Correspondence principle

Mach's principle

What is an inertial frame in Newtonian theory?



In this case we would have $\ddot{x} = \ddot{x}' + a$ and Newton's 2nd law suggests the presence of an inertial force in the accelerated system.

Typical examples are the centrifugal and Coriolis forces in a rotating system.

Always proportional to the mass.

What is the physical origin of inertial forces?

Newton: Motion relative to "absolute space".

Mach:

– All motion is relative
– Inertial frames are determined by fixed stars (mass distribution in the Universe)

Same effect if you rotate a bucket of water or keep the bucket fixed and "rotate the Universe".

1. Matter distribution determines the geometry.

2. No matter, no geometry.

3. A body in an empty Universe should have no inertial properties.

Mass in Newtonian theory

In classical physics we can distinguish between three different ways to "weigh" an object.

1. Inertial mass m^I measures "resistance" to acceleration;

$$\boldsymbol{F} = \frac{d(m^I \boldsymbol{v})}{dt} = m^I \boldsymbol{a}$$

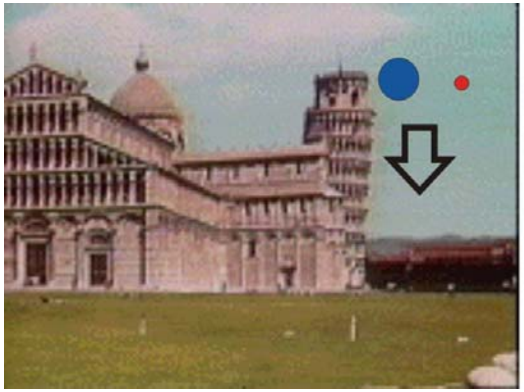
2. Passive gravitational mass m^P measures how the body is affected by a gravitational field;

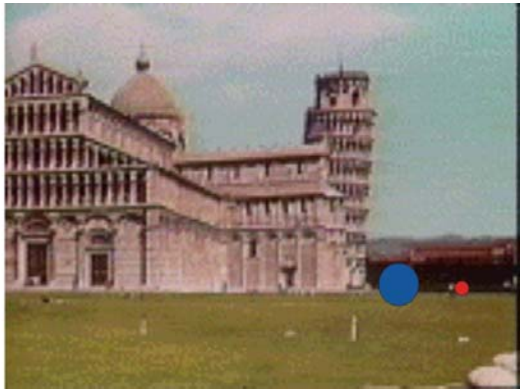
$$\boldsymbol{F} = -m^P \nabla \Phi$$

3. Active gravitational mass m^A measures the strength of the body as source of a gravitational field;

$$\Phi = -\frac{Gm^A}{r}$$

inertial = passive

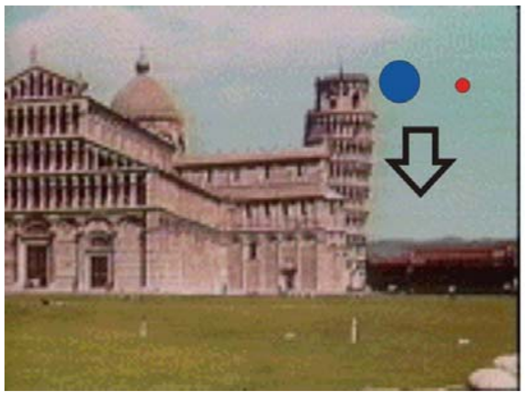


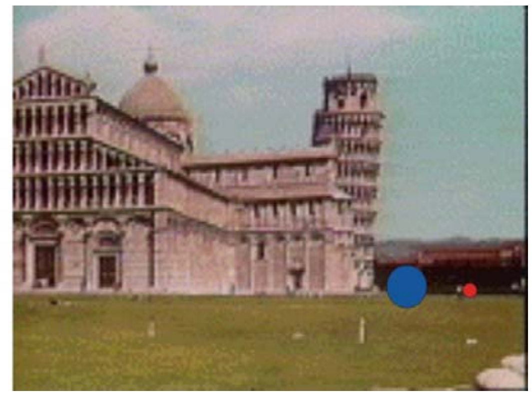


The story goes that in order to demonstrate to Aristotelian scholars that two balls of different weights fall at the same rate, Galilei dropped a cannon ball and a wooden ball from the top of the Tower of Pisa.

This story is apocryphal. However, when Galilei was an old man, one of his students did perform the demonstration and found a slight difference in the time the two balls struck the ground. This came as no surprise to Galilei who had already explained the effects of viscosity years before.

inertial = passive





$$m_1^I \boldsymbol{a}_1 = \boldsymbol{F}_1 = -m_1^P \nabla\Phi$$

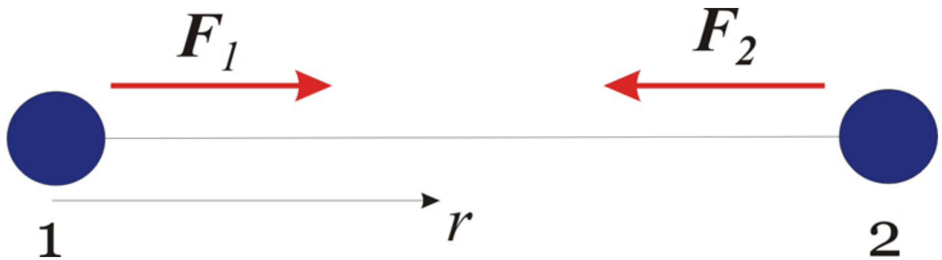
$$m_2^I \boldsymbol{a}_2 = \boldsymbol{F}_2 = -m_2^P \nabla\Phi$$

Observations demonstrate that $\boldsymbol{a}_1 = \boldsymbol{a}_2$, so it follows that

$$\frac{m_1^I}{m_1^P} = \frac{m_2^I}{m_2^P} = \alpha = 1 \quad \text{without loss of generality}$$

passive = active

Consider the gravitational attraction of two isolated bodies;



$$\Phi_1 = -\frac{Gm_1^A}{r} \qquad\qquad \Phi_2 = -\frac{Gm_2^A}{r}$$

Using Newton's 3rd law we see that

$$\left.\begin{aligned} \boldsymbol{F}_1 &= -m_1^P \nabla_Q \Phi_1 = \frac{Gm_1^P m_2^A}{r}\hat{\boldsymbol{r}} \\ \boldsymbol{F}_2 &= -m_2^P \nabla_R \Phi_2 = -\frac{Gm_2^P m_1^A}{r}\hat{\boldsymbol{r}} \end{aligned}\right\} \Rightarrow \frac{m_1^P}{m_1^A} = \frac{m_2^P}{m_2^A} = \beta = 1 \quad \text{w.l.o.g.}$$

$$\boxed{m^I = m^A = m^P = m}$$

Equivalence principle

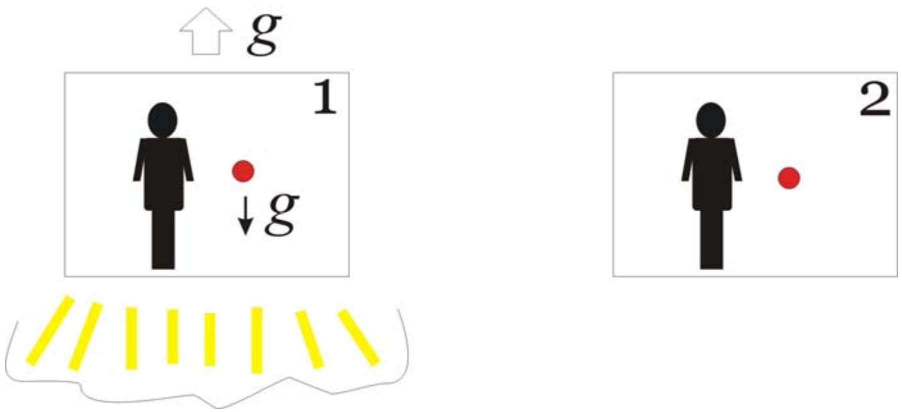
Strong: Motion of a test particle in a gravitational field is independent of its mass and composition.

Weak: Gravitational field couples to everything.

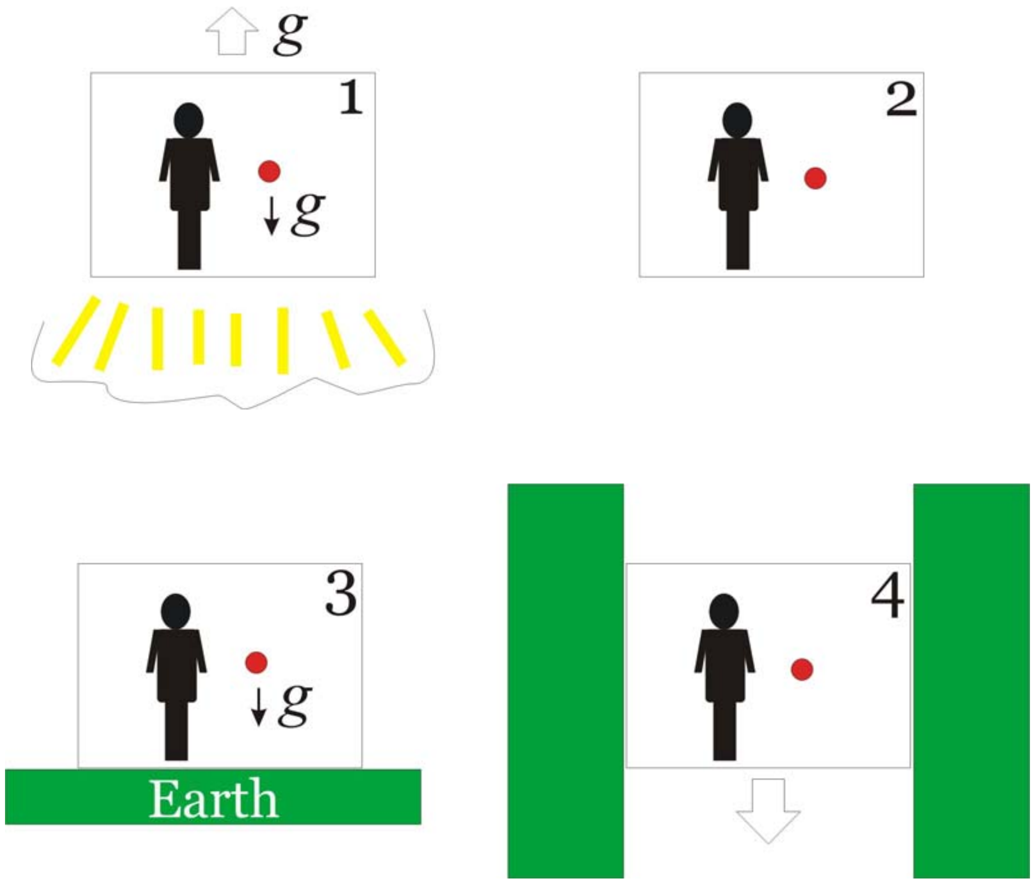
1. Local experiments can not distinguish free fall from uniform motion in space in absence of gravity.

2. A linearly accelerated frame can not be distinguished from a frame at rest in a gravitational field.

Lift experiments



Lift experiments



Gravity?

In a local inertial frame we can use Minkowski coordinates, which means that a test particle moves according to

$$\frac{d^2x^a}{d\tau^2} = 0$$

Meanwhile, in a non-inertial frame we must use general coordinates, leading to

$$\frac{d^2x^a}{d\tau^2} + \Gamma^a_{bc}\frac{dx^b}{d\tau}\frac{dx^c}{d\tau} = 0$$

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Inertial forces?

Einstein: Treat gravity as an inertial force. The metric g_{ab} is no longer flat.