The University Of Sheffield.

Department of Physics And Astronomy.

Spring Semester 2007 2 hours

GALACTIC DYNAMICS

Answer QUESTION ONE (COMPULSORY) and TWO others.

Each question is marked out of 10. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

A formula sheet and table of physical constants is attached to this paper.

NOTE:

Equations of motion in cylindrical polar coordinates R, ϕ, z, for an axisymmetric gravitational potential $\Phi(R, z)$:

$$-\frac{\partial \Phi}{\partial R} = \ddot{R} - R\dot{\phi}^2;$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(R^2\dot{\phi}\right) = 0;$$

$$-\frac{\partial \Phi}{\partial z} = \ddot{z}.$$

Standard values of constants:

Sun's distance from Galactic centre:	$R_0 = 8$ kpc
Velocity of Local Standard of Rest at R_0:	$V_0 = 215$ km s^{-1}
Oort's constants:	$A = 15$ km s^{-1} kpc^{-1}
	$B = -12$ km s^{-1} kpc^{-1}

1 **COMPULSORY**

(a) Show that the radial and transverse components, relative to the Sun, of the velocity of an object at distance *r* and Galactic longitude ℓ are given by

$$v_{\mathrm{R}} = (\Omega - \Omega_0) R_0 \sin \ell \qquad \text{and}$$
$$v_{\mathrm{T}} = (\Omega - \Omega_0) R_0 \cos \ell - \Omega r,$$

where Ω_0 and Ω are the angular velocities of the Sun and the other object respectively, and R_0 is the distance of the Sun from the Galactic centre. Assume all orbits are circular. [2]

(b) The equations of motion of a star in an axisymmetric gravitational potential $\Phi(R, z)$ can be written in terms of the effective potential

$$\Phi_{\mathrm{eff}}(R, z, L_z) = \Phi(R, z) + \frac{L_z^2}{2R^2},$$

where L_z is the *z* component of angular momentum. Show that the circular orbit of angular momentum L_z represents a minimum of this effective potential. [1]

Sketch the general form of Φ_{eff} as a function of *R*. Show on your graph the potential energy of a star in a nearly but not quite circular orbit, and explain *without mathematics* what this implies for the radial motion of the star. [1]

(c) In the context of theories of spiral structure, briefly explain

(i) the concept of *swing amplification*; [1]

(ii) the *Toomre stability parameter*, $Q = \dfrac{\sigma_R \kappa}{3.36 G \Sigma}$ (where the symbols have their usual meanings). [1]

(d) Estimate the mass density of the dark halo in the solar neighbourhood (in solar masses per cubic parsec), stating any assumptions that you make. Compare your result with the local disc density of about 0.15 $M_\odot$ pc^{-3}, and comment. [2]

(e) *N*-body simulations of galaxy formation in the standard scenario (dominated by cold dark matter and a cosmological constant) indicate that large galaxies like the Milky Way form from hierarchical mergers of many small clumps. In this context, discuss the observation that the Milky Way has ~10−20 dwarf galaxy satellites. [2]

2(a) Show that the mean time between close passages of the Sun and another star is ~10^{15} years. Define "close passage" as one which changes the Sun's kinetic energy by an amount comparable to its initial kinetic energy relative to the other star, and assume that the stellar number density in the solar neighbourhood is ~1 star per 10 pc^3, that the average mass of a star is ~0.5 solar masses, and that the velocities of disc stars in the solar neighbourhood are ~30 km s^{-1} relative to the Sun. [2]

(b) The Sun's velocity may also be perturbed by the combined effect of many weak, distant encounters. Show that the mean squared transverse velocity acquired by the Sun after a time *t* as a result of distant encounters with objects of mass *m* and number density *n* is

$$\left\langle V_T^2 \right\rangle = \frac{8\pi G^2 m^2 n t}{V} \ln\left(\frac{b_{\max}}{b_{\min}}\right),$$

where $b_{\min}$ and $b_{\max}$ are the minimum and maximum impact parameters considered, and *V* is the speed of the Sun relative to the other object. Justify the requirement that

$$b_{\min} >> \frac{2G\left(m + M_{\odot}\right)}{V^2}.$$ [3]

(c) Discuss the findings above in the context of

(i) field stars in our Galaxy; [1]

(ii) stars in open clusters, given that a typical open cluster is a few parsecs across and has relative stellar velocities of ~1 km s^{-1} stellar number densities of ~10 pc^{-3}; [1]

(iii) satellite galaxies orbiting within the halo of the Milky Way. [3]

3(a) By expanding $\Phi_{\text{eff}}\left(R,z,L_z\right)=\Phi\left(R,z\right)+\dfrac{L_z^2}{2R^2}$ in a Taylor series about its minimum at $R = R_C$, show that the (R, ϕ) position of a star in a nearly circular orbit is described by the equations

$$x = X\cos\left(\kappa t+\chi\right),$$
$$y = -\frac{2\Omega}{\kappa}X\sin\left(\kappa t+\chi\right),$$

where $x = R - R_C$, $y = R_C(\phi - \Omega t)$, Ω is the angular velocity of a star in a circular orbit at $R = R_C$, $\kappa^2 = -4B\Omega$, and X and are integration constants. [4]

(b) Stars in the solar neighbourhood have azimuthal velocities v measured relative to the Local Standard of Rest, *not* relative to their individual guiding centres. Show that under these circumstances $v = 2Bx$. [2]

Relative to the Local Standard of Rest, the Sun currently has $u = -10$ km s^{-1} and $v = 6$ km s^{-1}. Calculate the values of X and R_c for the Sun. [1.5]

(c) Explain what is meant by a *Lindblad resonance*. What is the relevance of Lindblad resonances to spiral structure? [1.5]

If the rotation curve of the Galaxy is approximately flat, and the ring of molecular gas at $R \sim 3$ kpc represents the inner Lindblad resonance of a two-armed spiral pattern, calculate the pattern speed and the location of the outer Lindblad resonance. [1]

4(a) The phase space number density $f(\mathbf{x},\mathbf{v})$ of stars must obey the continuity equation

$$\frac{\partial f}{\partial t}+\sum_{i=1}^{3}\left(\dot{r}_i\frac{\partial f}{\partial r_i}+\dot{v}_i\frac{\partial f}{\partial v_i}\right)=0\,.$$

Show that in cylindrical polar coordinates (R,ϕ,z), with corresponding linear velocities (U,V,W), this equation can be written in the form

$$U\frac{\partial f}{\partial R}+W\frac{\partial f}{\partial z}+\left(\frac{V^2}{R}-\frac{\partial\Phi}{\partial R}\right)\frac{\partial f}{\partial U}-\left(\frac{UV}{R}\frac{\partial f}{\partial V}\right)-\left(\frac{\partial\Phi}{\partial z}\frac{\partial f}{\partial W}\right)=0$$

for an axisymmetric steady state. [3]

(b) By multiplying this equation by W and integrating, show that

$$-\frac{\partial\Phi}{\partial z}=\frac{1}{N}\frac{\partial}{\partial R}\big(N\langle UW\rangle\big)+\frac{1}{N}\frac{\partial}{\partial z}\big(N\langle W^2\rangle\big)+\frac{\langle UW\rangle}{R}\,.$$ [3]

What physical property of the Galaxy might you measure using this equation? [1]

(c) What is an (isolating) *integral of the motion*? [1]

The only integrals of the motion for a general axisymmetric potential are the total energy E and the z-component of angular momentum L_z. Using Jeans' theorem, explain why this statement appears to contradict the observation that in the Galactic halo
$\sigma_{RR}=\sqrt{\langle U^2\rangle}=131\pm7\ \mathrm{km\,s^{-1}}$, $\sigma_{zz}=\sqrt{\langle W^2\rangle}=89\pm5\ \mathrm{km\,s^{-1}}$. [2]

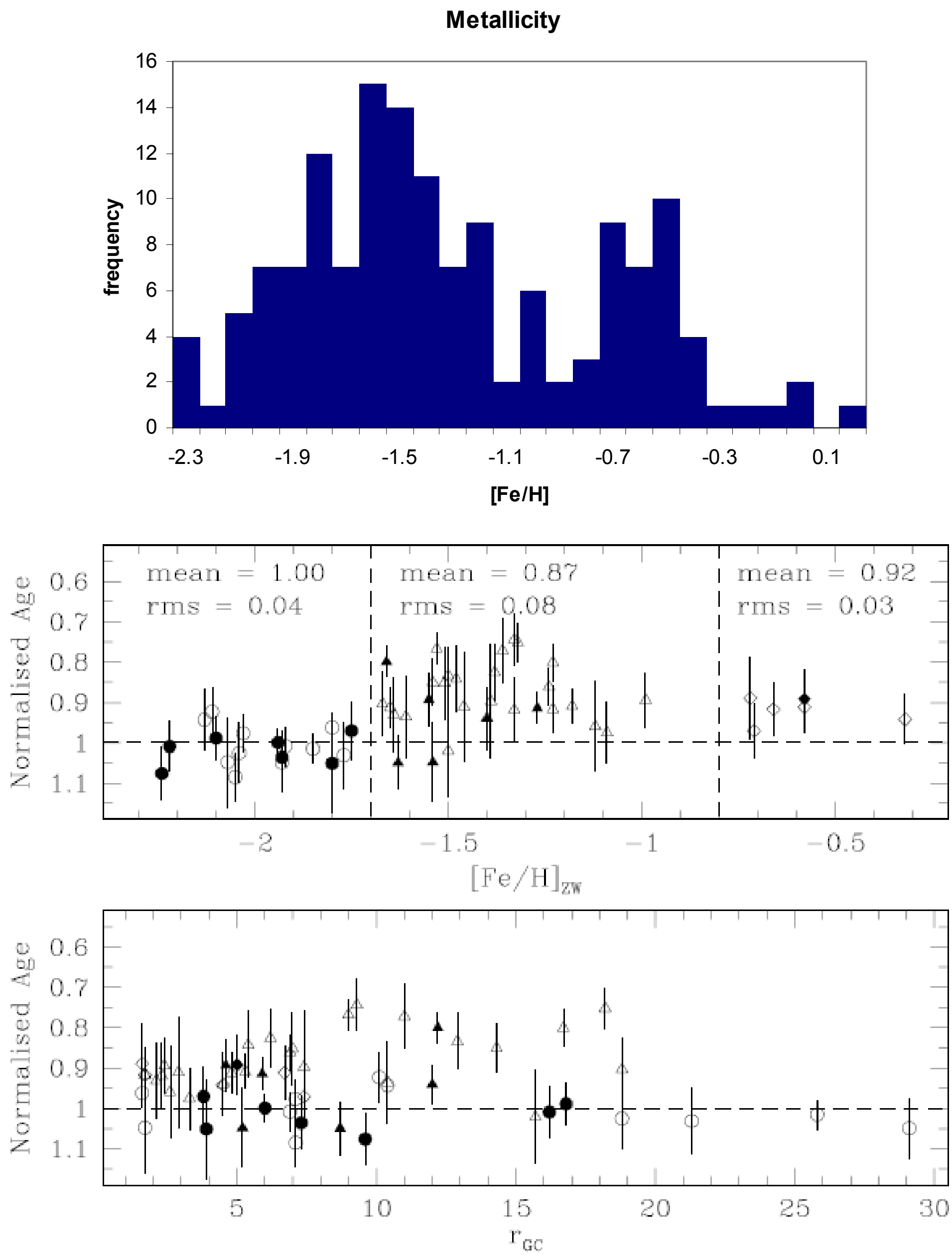
5(a) Discuss the observable signatures (in age, metallicity, and kinematics) expected if the stellar halo of the Milky Way was formed in

(i) a *rapid* dissipative collapse; [2]

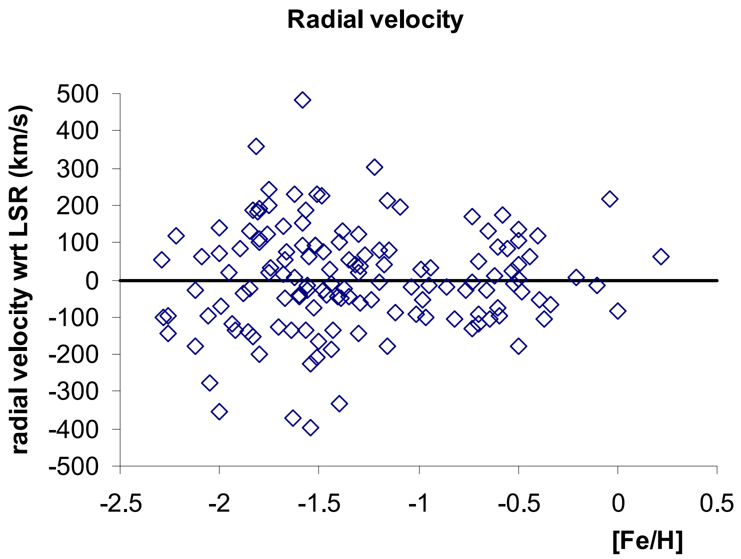
(ii) a *slow* dissipative collapse; [1]

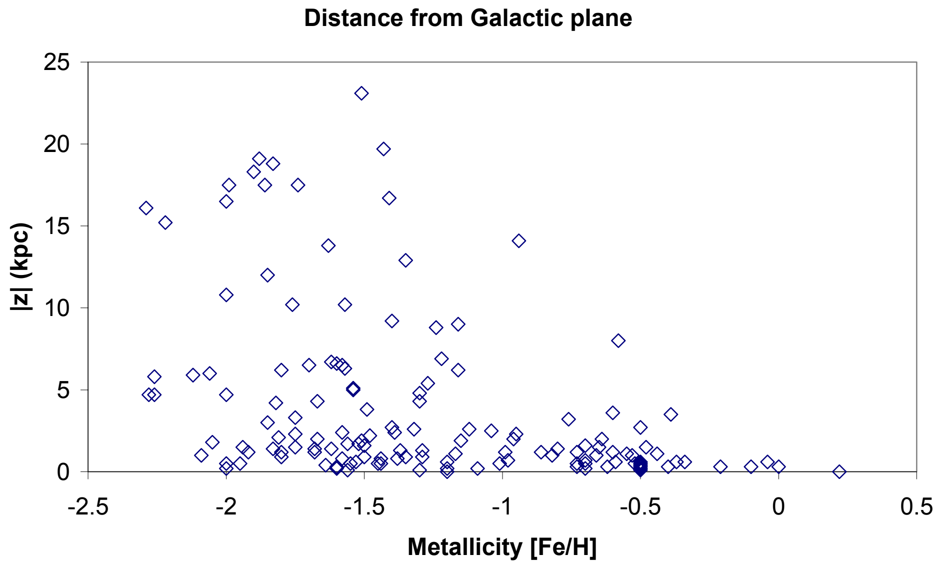
(iii) a series of hierarchical mergers. [2]

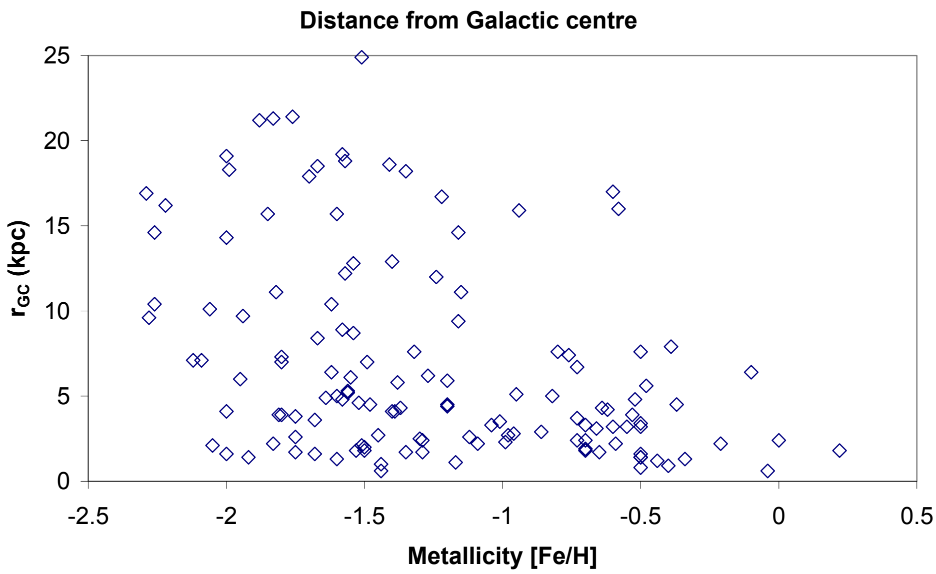
(b) The plots below and on the facing page show properties of the Galaxy's globular clusters. Using this information, discuss in detail the implications for the properties and evolution of the Galaxy's spheroidal systems. [5]



["Normalised age" is measured relative to the most metal poor clusters. Filled symbols are ground-based; open are HST. The shape of the symbol is defined by the metallicity.]







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