The University Of Sheffield.

DEPARTMENT OF PHYSICS & ASTRONOMY

Autumn Semester 2006-2007

MAGNETISM AND SUPERCONDUCTIVITY **2 Hours**

Answer THREE questions.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

1 (a) Explain how a diamagnetic material responds to an externally applied magnetic field H. Define the magnetisation M, the susceptibility χ and the relative permeability μ_r of a diamagnet and comment on their magnitudes. **[2]**

(b) What is the significance of the fact that all materials have a contribution to their susceptibility which is diamagnetic? **[1]**

(c) Describe the main features of the model that Langevin developed for the diamagnetic state. Hence show that the volume susceptibility χ of a diamagnetic material may be written

$$\chi = -\mu_0 NZ \frac{e^2}{6m} \left\langle \rho^2 \right\rangle ,$$

where N is the number of atoms per unit volume, Z is the atomic number, $\left\langle \rho^2 \right\rangle$ is the mean square radius of the electron orbits, μ_0 is the permeability of free space and e and m are the charge and mass of the electron respectively. **[5]**

(d) The mean square radius of the ground state electron orbit of hydrogen is $\left\langle \rho^2 \right\rangle = 3a_0^2$, where $a_0 = 4\pi\varepsilon_0 \hbar^2 / me^2$.

Use this information to calculate the molar diamagnetic susceptibility of a sample of atomic hydrogen gas. **[2]**

2 (a) Describe the fundamental features of the so-called “Molecular Field Model” which was proposed by P Weiss to explain the magnetic properties in the ferromagnetic state. **[2]**

(b) Hence explain, with an appropriate level of mathematical detail and the aid of sketch graphs where appropriate, how the spontaneous magnetisation of a ferromagnet may be calculated using the Molecular Field Model.

Your answer should make clear

(i) the significance of the Curie temperature T_C,

(ii) the role of the Brillouin Function $B_J(y)$ in your derivation.

It should include a graph of the reduced magnetisation $M(T)/M(0)$ versus the reduced temperature T/T_C. **[5]**

(c) In what circumstances does the Molecular Field Model fail to give a good description of the properties of a ferromagnet and why does this happen? **[2]**

(d) The Curie temperature of iron is $T_C = 1093$ K and each iron atom carries a magnetic moment of $2.22\,\mu_B$. Show that the magnitude of the Weiss Molecular Field must be of the order of 800 T. **[1]**

3 (a) Give an account, starting from the Sommerfeld free electron model of solids, of Stoner's itinerant electron theory of ferromagnetism.

Include sketch graphs in your answer, to show how a simple parabolic density of states curve is modified by the action of a magnetic field on the electrons.

Hence give a qualitative account of how Stoner's criterion for ferromagnetism

$$\gamma \mu_B^2 D(E_F) \geq 1$$

can be obtained.

Here γ is a molecular field constant and $D(E_F)$ is the density of (electronic) states at the Fermi surface. **[6]**

(b) What, briefly, is the form of the actual 3d and 4s electron bands in transition metals, and how does this help explain the absence of ferromagnetism in copper and its presence in nickel? **[2]**

(c) The notional molecular field in ferromagnetic iron is ≈ 700 T and the Fermi Energy approximately ≈ 7 eV.

Discuss whether these values suggest that the magnetic interactions represent a small perturbation at the Fermi Surface. **[2]**

4 In 1933, twenty-two years after the discovery of superconductivity, Meissner and Ochsenfeld studied the spatial variation of magnetic field around a superconductor.

(a) What was the nature of their observations? Describe with the aid of sketches the phenomenon known as the Meissner Effect. **[3]**

(b) Show that the laws of electromagnetism demonstrate that the Meissner Effect corresponds to a perfect diamagnetic state. **[2]**

(c) Explain what is meant by the critical field H_c of a superconductor and hence describe the division of superconductors into Type I and Type II examples.

Describe, with the aid of sketch graphs, the distinction between the magnetic properties of Type I and Type II superconductors. **[3]**

(d) The critical field of metallic tantalum (Ta) is $H_c(0) = 6.37 \times 10^4$ A m^{-1} at $T = 0$ K. Show by means of a sketch graph how the value of H_c will vary with temperature.

The superconducting transition temperature of tantalum is $T_c = 4.48$ K. What is the value of H_c at 2.24 K? **[2]**

5 The London Equation (below) arises from a description of the Meissner Effect in terms of superconducting surface currents, which screen the interior of a superconducting sample,

$$\nabla\times\mathbf{j} = -\frac{n_s e^2}{m}\mathbf{B}.$$

Here n_s is the number density of superconducting electrons, e and m are the charge and mass of the electron respectively and $\mathbf{B}$ is the magnetic induction field.

(a) Show how the London equation can be used with Ampère's theorem,

$$\nabla\times\mathbf{H} = \mathbf{j} + \frac{\partial\mathbf{D}}{\partial t} \ ,$$

to demonstrate that an external magnetic field will decay exponentially with distance into the surface of a superconductor. **[5]**

You may use the identity $\nabla\times(\nabla\times\mathbf{A}) = \nabla(\nabla\cdot\mathbf{A}) - \nabla^2\,\mathbf{A}$ where $\nabla\times\mathbf{j} = curl\,\mathbf{j}$ and $\nabla\cdot\mathbf{A} = div\,\mathbf{A}$.

(b) Hence show that the London penetration depth λ_L is given by the expression

$$\lambda_L = \frac{1}{e}\sqrt{\frac{m}{n_s\,\mu_0}} \ ,$$

where μ_0 is the permeability of free space.

Calculate the value of London penetration depth λ_L in a metal where the number density of superconducting electrons is $n_s = 1 \times 10^{29}$ m^{-3}.

[2]

(c) How do you expect the London penetration depth λ_L to vary with temperature T, and how can this variation be simply explained? **[3]**

END OF EXAMINATION PAPER