The University Of Sheffield.

DEPARTMENT OF PHYSICS & ASTRONOMY

Autumn Semester 2006-2007

METALS, SEMICONDUCTORS AND INSULATORS **2 Hours**

Answer THREE questions.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

1 (a) Give an account, with reference to a structural prototype, of the main features of metallic bonding.

Hence, explain which crystalline structures metals may be expected to adopt and the nature of any relationships between them. **[5]**

(b) There is a qualitative description of metallic bonding, often found in text books, stating that

"the ion cores are washed in a conduction electron sea".

Use the information given below for metallic lithium (Li), which has a body centred cubic structure, to indicate whether this is a reasonable description.

The unit cell dimension of lithium is a = 0.3491 nm, the Goldschmidt radius of a lithium atom r_G = 0.151 nm and the radius of a Li^+ lithium ion r_{Li+} = 0.078 nm. **[1.5]**

(c) Lithium and most of the other metals in the Periodic Table, appear "silvery" in colour. What physical mechanism is responsible for this and what is the relevance of "plasma oscillations" in determining the colour of metals?

The so-called plasma frequency ω_p of a metal is given by the equation

$$\omega_p = \sqrt{\frac{n e^2}{\varepsilon_0 m}} \ ,$$

where n is the electron density ε_0 is the permeability of free space and e and m are the electronic charge and mass respectively.
What is the value of ω_p for metallic lithium?

How does this help explain its silvery colour? **[3.5]**

2 In the questions below, include appropriate diagrams and sketches to support your answers.

(a) Give an account of Sommerfeld's free electron theory of solids.

Explain the concept of the Fermi sphere and hence obtain the expression for the wavevector k_F at the Fermi surface,

$k_F = \sqrt[3]{3\pi^2 n}$, where n is the electron density.

Use this result to derive an expression for the Fermi energy E_F. Rubidium (Rb) is a simple metal with a single conduction electron ($5s^1$) and a body centred cubic structure with a unit cell dimension $a = 0.5585$ nm. What are the values of k_F and E_F for rubidium? **[5]**

(b) In a real metal with a crystalline lattice, there are discontinuities in the k values which define the boundaries of the first and subsequent Brillouin zones.

What happens, and why, in a real metal if the electron density n is increased to a point where the Fermi surface approaches the first Brillouin zone boundary?

What is the experimental evidence for this behaviour? **[3]**

(c) When a metal has a large electron density n, the energy levels in the second and higher order Brillouin zones will also be partially filled.

Show how mapping the Fermi surface in the higher order zones can help to identify "electron-like" and "hole-like" sections of the Fermi surface. **[2]**

3 (a) Make a properly labelled sketch of the electron energy bands $E(k)$ *versus* k for an intrinsic semiconductor, using the reduced zone representation.

Identify the valence band(s) and the conduction band(s) and indicate which you expect to be populated when the semiconductor is at $T = 0$ K.

What is a typical value for the energy gap E_g of an intrinsic semiconductor?

Describe with the aid of a further sketch, or sketches, what you understand by the terms direct transition and indirect transition, and explain what is involved in each case. **[3]**

(b) The actual mass m of an electron moving in a periodic potential may be replaced by the effective mass m^*.

(i) If the electron velocity v is equal to the group velocity of the electron waves then, show that

$$v = \frac{1}{\hbar}\frac{\mathrm{d}E}{\mathrm{d}k},$$

[1]

(ii) and that the application of a force F produces a change in the motion of an electron which is described by

$$F = \hbar\frac{\mathrm{d}k}{\mathrm{d}t}\ .$$

[1]

(iii) Hence, derive the expression below, for the effective mass m^*, by considering the acceleration of the electron which is produced by the action of the force F.

$$\frac{1}{m^*} = \frac{1}{\hbar^2}\frac{\mathrm{d}^2E}{\mathrm{d}k^2}\ .$$

[2.5]

(c) Use the identities given above to sketch the variation of the electron velocity v and the effective mass m^* with the wave vector k across the first Brillouin zone of a crystalline solid and describe the nature of the variations predicted. **[2.5]**

4 The experimental methods used to study the properties of electrons in solids are often based on the simultaneous application of electric and magnetic fields.

(a) Explain what is meant by "cyclotron resonance" and derive the expression for the cyclotron frequency $\omega_c = \frac{eB}{m^*}$, where e and m^* are the charge and effective mass of the electron and B is the magnetic induction field. **[3]**

(b) Describe, with the aid of a diagram, a typical experimental arrangement that is used in cyclotron resonance, indicating the method of operation and the physical principles involved.

Explain what requirements the sample must satisfy in order that resonances can be successfully observed. **[2]**

(c) Sketch a graph of a typical data set obtained in a cyclotron resonance experiment and hence explain how electrons and positive holes can be distinguished and how carriers with different effective masses m^* can be identified.

What information can be obtained from the linewidth of the resonances? **[3]**

(d) A cyclotron resonance experiment is performed on an *n*-type semiconductor at a frequency of 180 GHz, in a magnetic field of 1.7 T.

Find the effective mass of the electrons.

The mean time $\langle \tau \rangle$ between the electron collisions must be greater than what value if well resolved resonances are to be observed? **[2]**

5 (a) A bulk specimen of a dielectric material is placed in a uniform electric field. Explain what is meant by the Electric Displacement D, the Polarisation P, and the electric susceptibility χ and give the equations which relate these quantities. **[2]**

(b) Given that there are no free carriers in the dielectric material, what mechanism can be proposed, at an atomic level, to account for the bulk Polarisation P?

How can the bulk Polarisation P be expressed in terms of an atomic polarizability α? **[2]**

(c) Describe an atomic scale model, and give an outline of the theoretical treatment, that can be used to show that for a spherical atom in a gas, the atomic polarizability α is directly related to the atomic volume through the equation

$$\alpha = 3\varepsilon_0 \left[\frac{4}{3} \pi r^3 \right] \quad .$$

[4]

(d) Hence find the value of the atomic polarizability of helium (in units of F m^2) which has an atomic radius of $r = 0.102$ nm. **[2]**

END OF EXAMINATION PAPER