The University Of Sheffield.

DEPARTMENT OF PHYSICS & ASTRONOMY

Spring Semester 2006-2007

TECHNIQUES OF PROBLEM SOLVING **2 Hours**

Marks will be awarded for a candidate's SIX best answers.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten.

1 An uninsulated metal tank contains refrigerant which is maintained at –12°C. A layer of ice grows on the outside of the tank until equilibrium is reached when conduction through the ice balances the heat received by radiation from the surroundings, which are at 20°C. Estimate the thickness of the ice at equilibrium.

Specific heat of ice is 2.1×10^3 J kg^{-1} K^{-1}; latent heat of fusion of ice is 3.3×10^5 J kg^{-1}; thermal conductivity of ice is 2.3 W m^{-1} K^{-1}.

2 A pygmy shrew has a mass of about 3.5 g and has to eat its own body weight of food daily to maintain its body temperature. What fraction of its body weight (6000 kg) must an elephant eat?

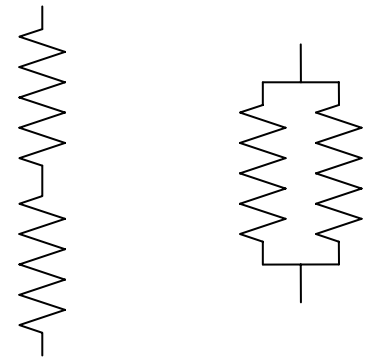
3 A project calls for a bi-convex lens with a temperature independent focal length. In a search for possible glasses, the designer considers one with a room temperature refractive index of 1.45 and a temperature coefficient of refractive index of 3×10^{-5} per degree. What thermal expansion coefficient must the glass have to meet the specification?

Note: the lens makers' formula for the focal length f of a symmetrical biconvex lens is $f = \dfrac{R}{2(n-1)}$, with R the radius of curvature and n the refractive index.

4 A snooker ball of radius R is struck by a horizontally moving cue. Find the cue height (above the table) such that no spin occurs and the ball just rolls away across the table.

Note: the moment of inertia of a sphere of mass M is $\frac{2}{5}MR^2$.

5 What is the spring constant of two springs each of spring constant k connected one on the end of the other (in series)? What is the spring constant for a load attached to the end of the two springs connected side by side (in parallel)?



series parallel

6 A rollercoaster slides down a frictionless track and then enters a vertical loop of radius r. Show that it must be released from a minimum height of $5r/2$ above the bottom of the loop if it is to remain in contact with the loop at all points.

7 A bucket of water is sliding freely down an inclined plane at an angle θ to the horizontal. Assuming that the contact between the bucket and the plane is completely frictionless, determine the shape and orientation of the water surface.

8 What is the electrostatic self-energy of a spherical drop of radius R carrying a total charge Q spread uniformly over its surface? If surface tension leads to an energy per unit surface area of σ, what is the critical charge Q_C above which the drop will be unstable and break into two smaller droplets – a phenomenon known as *Rayleigh instability*?

9 Three charges of +1, +2 and +1 C are placed in a line. The two smaller charges are fixed at a distance $2R = 2.0$ mm apart with the larger charge halfway between, but able to move along the line joining, the other two charges. The mass of the larger charge is 30 g. What wavelength electromagnetic radiation can this system absorb?

10 Core-collapse supernovae (exploding massive stars) release almost all of their energy in the form of neutrinos of energy ~10 MeV. Neutrinos are now known to have a very small mass $m \sim 1$ eV or less.

A supernova has exploded at a distance $d = 1.5 \times 10^{21}$ m from us, producing an instantaneous burst of neutrinos. I have a neutrino detector which can measure the arrival times and energies of incoming neutrinos, and I find that the first neutrino in the supernova signal has an energy of 20 MeV, while the last arrives 12.4 s later with an energy of 8.9 MeV.

What is the mass of the neutrinos produced in this explosion?

11 In 2006, a man was apparently killed by radiation poisoning about 10 days after ingesting Polonium-210. Given that the lethal dose is estimated as 5 Grey (i.e. 5 J kg^{-1}) and that ^{210}Po emits alpha particles of energy 5.3 MeV with half-life 138 days, estimate the minimum amount of polonium which must have been taken into his body.

12 A particle of mass m is bound in a one-dimensional potential and has an unnormalised wavefunction

$$\psi(x) = \left(\frac{x}{a}\right)^2 e^{-\left(\frac{x}{a}\right)^2}$$

with energy $E = \dfrac{5\hbar^2}{ma^2}$. What is the potential that binds the particle? Which bound state does the wavefunction represent?

13 A sample of gas is allowed to expend adiabatically. During the process its pressure drops from 120 to 100 kPa, with a corresponding temperature drop from 27°C to 6°C. Calculate $\gamma = C_p / C_v$. Is the gas monatomic or diatomic?

($C_p = C_v + R$ for one mole.)

END OF QUESTION PAPER

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