The University Of Sheffield.

DEPARTMENT OF PHYSICS & ASTRONOMY

Autumn Semester 2006-2007

STELLAR ATMOSPHERES **2 Hours**

Answer THREE QUESTIONS.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

1. (a) For an absorption line, define the line depth R_λ and equivalent width W_λ. Sketch the line profile for an optically thin and an optically thick line. Does LTE hold for the line core or wings of optically thick lines? [2]

(b) Discuss the following broadening mechanisms for absorption lines, including a discussion of their associated line profiles:

i. Doppler broadening;

ii. natural broadening;

iii. pressure broadening.

What is a Voigt profile? [2.5]

(c) The Si III (atomic mass 28) line at 456.7 nm is observed in a B star with an effective temperature of T=17,500 K.

i. The full width at half maximum (FWHM) for a thermally broadened spectral line, $\Delta\lambda^{\rm D}_{1/2}$ in nm, is given by

$$\frac{\Delta\lambda^{\rm D}_{1/2}}{\lambda} = 7.16 \times 10^{-7}\sqrt{\frac{T}{\mu}}$$

where T is the temperature in K, λ is the line wavelength in nm, and μ is the atomic mass in atomic mass units. Calculate the thermally broadened FWHM of this line. [1]

ii. The measured FWHM greatly exceeds the predicted thermal width from part (i). Which other broadening mechanism might be responsible for this observed FWHM? [0.5]

iii. Would your answers to parts (i) and (ii) be the same, if instead we were considering the Hγ at 434.0 nm in the B star? [1]

(d) What is the curve of growth, and how can it be used to determine elemental abundances in stellar photospheres? Explain the dependence of the three distinct parts of the curve of growth on the number density. Show diagramatically examples of line profiles in each of these domains. [3]

2. (a) The Saha equation may be written in the form

$$\frac{N^+ n_e}{N} = C \frac{u^+}{u} T^{3/2} \exp\left(-\frac{\chi}{kT}\right)$$

where $C = 4.83 \times 10^{21}\,\mathrm{m}^{-3}$. Discuss its importance in the study of stellar atmospheres. Carefully explain the meaning of each term and explain how u, u^+ can be obtained. [2]

(b) An O-type star with a pure helium photosphere has a surface temperature of 40,000 K and an electron pressure of 1000 $\mathrm{N\,m^{-2}}$.

i. Determine the electron density, and hence calculate the ratio of doubly to singly ionized helium. [2.5]

ii. What is the ratio of electron to gas pressure in the O star, assuming a negligible fraction of neutral helium? [1.5]

(c) i. Define surface gravity, and hence solve the equation of hydrostatic equilibrium

$$\mathrm{d}P_g(r) = \frac{GM(r)\rho(r)}{r^2}\mathrm{d}r$$

for a plane-parallel atmosphere, where $P_g(r)$ is the gas pressure, and $\rho(r)$ is the density, i.e. derive

$$\frac{\mathrm{d}P_g(r)}{\mathrm{d}r} = g\rho(r) \qquad \text{where} \qquad \rho(r) = \frac{\mu(r) m_{\mathrm{H}}}{kT} P_g(r)$$

and $\mu(r)$ is the mean molecular weight. [1.5]

ii. For an isothermal atmosphere with a constant mean molecular weight, integrate the equation from part (i) from r_0 to r to obtain

$$P_g(r) = P_g(r_0) \exp[(r - r_0)/H]$$

if we introduce the scale height $H = kT/(g\,\mu\,m_{\mathrm{H}})$. Explain the physical significance of H, and derive its value for the solar case, assuming a pure hydrogen atmosphere at $T = 5777\,\mathrm{K}$. [2.5]

Note: The ionization energy of ionized helium is 54.4eV. The partition functions for singly ionized and doubly ionized helium are 2 and 1, respectively.

3. (a) i. The transfer equation for a plane-parallel stellar atmosphere is

$$\cos\theta \frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda$$

Define each term in this equation, and derive the equation for the surface intensity

$$I(0,\theta) = \int_0^\infty S_\lambda e^{-\tau_\lambda \sec\theta} d(\tau_\lambda \sec\theta). \quad [2.5]$$

ii. If we adopt a linear source function of the form,

$$S_\lambda(\tau_\lambda) = a_\lambda + b_\lambda \tau_\lambda,$$

show that

$$I(0,\theta) = S_\lambda(\cos\theta). \quad [1.5]$$

You may use the standard integral

$$\int_0^\infty u^n e^{-u} du = n!$$

iii. Using the above relationship, or otherwise, explain the concept of limb darkening in stellar atmospheres. [1.5]

(b) i. Explain how limb darkening observations can be used to obtain the optical depth dependence of the temperature of a stellar photosphere. How does this help identify the source of continuous opacity in stellar atmospheres? [2.5]

ii. Give two methods used to derive limb darkening information for stars other than the Sun. [2]

4. (a) Define Local Thermodynamic Equilibrium (LTE). Provide an example of an astrophysical situation in which it is necessary to consider non-LTE. [1.5]

(b) Energy levels of hydrogen lie $13.6/n^2$ eV below the ionization limit. Calculate the threshold wavelengths of the Balmer and Paschen continua. [1.5]

(c) Consider a pure hydrogen stellar photosphere with effective temperature 7,500 K and electron pressure of 10 N m^{-2}.

i. Assuming LTE, use the Saha and Boltzmann equations to compare the relative number of H atoms and H^- ions which contribute to the continuous opacity on both sides of the Paschen jump. [2]

ii. Which absorption or scattering process provides the majority of the continuous opacity at wavelengths shortward, and longward, of the Paschen jump? You may adopt identical bound-free cross-sections for atomic hydrogen and H^- for simplicity. [2]

iii. Is the strength of the Paschen jump in this star sensitive to temperature and/or pressure? Explain your answer. [1.5]

(d) For main-sequence stars, the strength of the Balmer jump is observed to be weak in late-type stars, strong in A and B type stars, and weak in O stars. Explain this behaviour. [1.5]

Note: The ionization energy for the negative hydrogen ion is 0.75eV and neutral hydrogen is 13.6eV. Saha's equation is

$$\log \frac{N^+}{N} = \log \frac{u^+}{u} + \frac{5}{2} \log T - \frac{5040}{T} \chi - \log P_e - 1.176$$

5. (a) By considering the solution to the parallel ray transfer equation

$$I_\lambda = I_{\lambda 0} e^{-\tau_\lambda} + S_\lambda (1 - e^{-\tau_\lambda})$$

for an incident intensity $I_{\lambda 0}$, explain why an emission line spectrum results from an ionized nebula, whilst stellar photospheres generally produce an absorption line spectrum. [3]

(b) i. Define the term 'grey atmosphere'. Identify one form of continuous opacity in stellar photospheres which is grey. [1]

ii. The Eddington approximation is $K_\lambda = J_\lambda/3$, where

$$K_\lambda = \frac{1}{4\pi}\int I_\lambda \cos^2\theta \, d\omega \qquad \text{and} \qquad J_\lambda = \frac{1}{4\pi}\int I_\lambda \, d\omega.$$

Assuming the Eddington approximation, show for a grey atmosphere in LTE that the total radiation pressure P_R is equal to

$$P_R = \frac{4\sigma}{3c} T^4$$

where the radiation pressure, P_λ, at wavelength λ is

$$P_\lambda = \frac{1}{c}\int I_\lambda \cos^2\theta \, d\omega.$$ [2]

(c) i. Briefly describe how radiation pressure drives stellar winds in early-type stars, and identify two observational diagnostics of mass-loss from such stars. [2]

ii. The Eddington parameter, Γ_e, can be written as

$$\Gamma_e = 10^{-4.5} q \frac{L/L_\odot}{M/M_\odot},$$

where q is the number of free electrons per atomic mass unit. Explain the meaning of Γ_e and derive the Eddington luminosity for a completely ionized helium atmosphere of mass $20 M_\odot$. [2]

END OF QUESTION PAPER