The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Autumn 2006-2007

STELLAR STRUCTURE AND EVOLUTION **2 HOURS**

Answer THREE questions.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

TURN OVER

1. In a star of mass M_s, the density decreases from the centre to the surface as a function of radial distance r, according to

$$\rho = \rho_c \left[1 - \left(\frac{r}{r_s} \right)^3 \right],$$

where ρ_c is the central density (and is a given constant) and r_s is the star's radius.

Show that the mass of the star is given by

$$M_s = \frac{2\pi\rho_c r_s^3}{3}.$$ [2]

Hence show that the gravitational potential energy Ω of the star is given by

$$\Omega = -\frac{153GM_s^2}{220r_s}.$$ [5]

Assuming that the Sun has the density profile given above and is devoid of nuclear energy sources, at what rate would it have to shrink (in metres per year) to maintain its present luminosity? [3]

2. A spherical star is in hydrostatic equilibrium. Assume that it is composed of an ideal gas of uniform mean particle mass, that there is negligible radiation pressure, and that the gas pressure vanishes at its surface. Prove the following results:

(a) $P_c > \dfrac{GM_s^2}{8\pi r_s^4}$, [3.5]

(b) $\overline{T} > \dfrac{GM_s m}{6kr_s}$. [4.5]

P and T are the pressure and temperature at radius r, M the mass contained within radius r, and the suffixes c and s refer to central and surface values. $\overline{T}$ is the mean temperature defined by

$$M_s \overline{T} = \int_0^{M_s} T \, \mathrm{d}M.$$

G, k and m are the gravitational constant, Boltzmann's constant and the mean particle mass, respectively.

Use relation (b) to determine the minimum mean temperature of the Sun and then show that the assumption of negligible radiation pressure is valid at a typical point in the Sun. [2]

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3. Explain what is meant by the term '*polytrope*'. [1]

Supposing that a spherical star has an equation of state

$$P = K\rho^2,$$

where K is a constant, use the equations of *hydrostatic support* and *mass conservation* to show that

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2\frac{\mathrm{d}\theta}{\mathrm{d}\xi}\right)+\theta = 0, \qquad \text{(equation 1)}$$

where $r = \alpha\xi$, $\alpha = \sqrt{K/2\pi G}$ and $\rho = \rho_c\theta$. P and ρ are the pressure and density at radius r and G is the gravitational constant. The suffix c refers to the central values of the variable. [6]

Verify that equation 1 has the solution

$$\theta = \frac{\sin\xi}{\xi}.$$ [3]

4. (a) Give a brief account of the various mechanisms which contribute to the *opacity* of stellar material. [3]

(b) State, giving reasons, whether conduction or radiation is more important in transporting energy inside main sequence stars. [2]

(c) By considering the effect of a small displacement upon an element of stellar material, derive the following expression for the minimum temperature gradient required for convection to occur in a stellar interior,

$$\frac{P}{T}\frac{\mathrm{d}T}{\mathrm{d}P} > \frac{\gamma-1}{\gamma},$$

where P is the pressure, T the temperature, and γ the ratio of specific heats. [5]

5. Describe the evolution of a typical star of solar metallicity and one solar mass from the time it joins the main sequence to the cessation of nuclear fusion. Diagrams should be used where appropriate. [10]

END OF QUESTION PAPER