DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring Semester 2006-2007

ATOMIC SPECTRA AND RELATIVITY **2 HOURS**

Answer THREE questions including at least ONE from each of sections B & C.

*Answers to different sections **must** be written in separate books, the books tied together and handed in as one.*

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

The Lorentz transformations for space co-ordinates x, y, z and time co-ordinate t are

$$x' = \gamma(x - vt) \quad y' = y \quad z' = z \quad t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

for relative motion with velocity v along the x-axis.
Here γ is the Lorentz gamma factor defined by

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}.$$

TURN OVER

SECTION A

1.

(a) An asteroid of length 100 m travels at speed $0.5c$ past a stationary observer. What is the observed length of the asteroid as it passes the observer? [1]

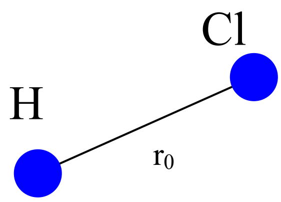
(b) What is the total energy in MeV of an electron travelling at a speed $0.9c$? [1]

(c) Monoenergetic cosmic ray muons are created at an altitude of 10 km above the earth's surface. The muons travel with constant velocity vertically downwards. Assuming that, on average, the muons reach the earth's surface before decaying, derive a lower limit on

(i) the muons' speed (as a fraction of c), and hence
(ii) the muons' energy (in MeV).

You may assume that the rest energy of the muons is 105.6 MeV and their mean lifetime at rest is 2.2×10^{-6} s. [3]

(d)



An HCl molecule with angular momentum L and moment of inertia I has a rotational energy that can be written as $E = \frac{L^2}{2I}$. The moment of inertia is given by $I = \mu \cdot r_0^2$, where $\mu = \frac{m_H m_{Cl}}{m_H + m_{Cl}}$ is the reduced mass and r_0 is the distance between the masses.

Assuming that angular momentum is quantized according to $L = \sqrt{l(l+1)}\hbar$,

(i) find an expression for the energy ΔE_l needed for a transition from an energy level $l-1$ to an energy level l, where l is the orbital angular momentum quantum number; [2]

(ii) if a transition from the level $l=1$ to the level $l=2$ requires $\Delta E_{l=2} = 0.0077$ eV, calculate the intermolecular distance r_0. You may take $m_H = 938\,\text{MeV}/c^2$ and $m_{Cl} \approx 35 m_H$ and you may find it helpful in your calculation to use the relation $\hbar c = 1240/2\pi$ eV nm. [3]

SECTION B

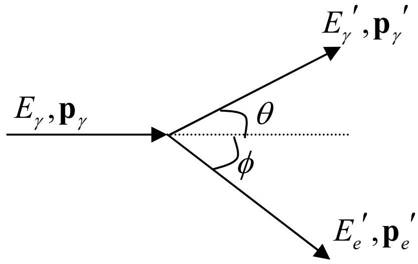
2.

(a) Use the relativistic expressions for energy E and for momentum $\mathbf{p}$,

$$E = m_0 \gamma c^2, \qquad \mathbf{p} = m_0 \gamma \mathbf{v},$$

where m_0 is the rest mass and γ is the Lorentz gamma factor for velocity v, to show that

$$E^2 = \mathbf{p}^2 c^2 + m_0^{\,2} c^4 .$$ [2]



In Compton Scattering, a high energy photon of initial energy E_γ scatters from a stationary electron of mass m_e through an angle θ, obtaining a final energy E_γ' (see diagram above). The electron recoils through an angle ϕ with energy E_e'.

(b) Show by considering the conservation of energy that

$$\left(E_\gamma - E_\gamma'\right) + m_e c^2 = E_e' .$$ [1]

(c) Show by considering the conservation of momentum that

$$\begin{aligned} E_\gamma - E_\gamma' \cos\theta &= p_e' c \cos\phi \\ -E_\gamma' \sin\theta &= p_e' c \sin\phi . \end{aligned}$$ [2]

(d) Using the results of (c) show that

$$E_\gamma^{\,2} + E_\gamma'^2 - 2E_\gamma E_\gamma' \cos\theta = p_e'^2 c^2$$ [1]

(e) Use the results of (b) and (d) to show that

$$\frac{1}{E_\gamma'} - \frac{1}{E_\gamma} = \frac{(1-\cos\theta)}{m_e c^2}$$ [3]

(f) A gamma ray photon of energy 1 MeV is Compton scattered through an angle of 60°. What is the energy of the photon after scattering? [1]

3.

(a) The relativistic transformations for energy and momentum are given by

$$p_x' = \gamma\left(p_x - \frac{v}{c^2}E\right) \quad \text{and} \quad E' = \gamma\left(E - vp_x\right).$$

A source moves with speed v in the rest frame of an observer and emits a photon of frequency ν at angle θ to its direction of motion as measured in its rest frame. Using the results above show that the frequency ν' and emission angle θ' of the photon measured by the observer are given by

$$\nu' = \gamma\nu(1-\beta\cos\theta) \ ,$$

$$\tan\theta' = \frac{\sin\theta}{\gamma(\cos\theta-\beta)} \ ,$$

where $\beta = \dfrac{v}{c}$ and $\gamma = \left(1-\beta^2\right)^{-\frac{1}{2}}$. [4]

(b) Hence show that for a given emission angle θ the minimum observed frequency is obtained when $\beta = \cos\theta$.
[You need not prove that the frequency is at a minimum rather than a more general turning point.] [3]

(c) A spaceship signals an observer by firing a collimated laser beam at $\theta = 60^{\circ}$ to its direction of travel. At what speed must the ship pass the observer to minimise the energy of the beam they receive? [1.5]

(d) What is the apparent direction θ' of the laser beam relative to the direction of travel of the spaceship, as measured by the observer when receiving the pulse? [1.5]

CONTINUED

SECTION C

4.

The Bohr radius α_0 and the Rydberg energy E_R for the hydrogen atom are given by

$$\alpha_0 = \frac{4\pi\varepsilon_0}{e^2}\frac{\hbar^2}{m_e}\ , \qquad E_R = \frac{e^2}{8\pi\varepsilon_0\alpha_0} = 13.6\ \text{eV}.$$

Consider an electron with principal quantum number n orbiting a point charge Ze with an effective potential, given as a function of r by

$$V(r) = -\frac{Ze^2}{4\pi\varepsilon_0 r} + \frac{l(l+1)\hbar^2}{2m_e r^2},$$

where l is the total angular momentum quantum number.

(a) By minimising the effective potential, derive an expression for the modified Bohr radius α_0' as a function of α_0. [2]

(b) Using your result from (a), show that the ionization energy E_I of the electron (the energy needed to remove an electron from the atom) is given by $E_I = Z^2 E_R$. [3]

(c) Hence calculate the ionization energies of the one-electron ions He^+ and Li^{2+}. [2]

(d) In the sodium atom, the valence electron is partially screened from the full nuclear charge of $11e$ by the other 10 electrons in the atom. To a first approximation we may therefore define the electron–nuclear Coulomb interaction with an effective nuclear charge Z^*e where $Z^* < 11$. The binding energy of the valence electron is found to depend on its angular momentum state as follows:

5.12 eV for the 3*s* state
2.10 eV for the 3*p* state and
1.52 eV for the 3*d* state.

Calculate the effective point charge Z^*e in each case and, considering the trend observed for 3*s*, 3*p* and 3*d* states, explain why an electron in different angular momentum states experiences different values of Z^*e. [3]

TURN OVER

5.

(a) Explain concisely why the 2*s* state of the hydrogen atom cannot decay by electric dipole radiation to the 1*s* state. [2]

(b) Explain why up to 10 electrons may be assigned to the 3*d* orbitals and why 14 electrons can be assigned to 4*f* orbitals. [2]

Consider the following four electron configurations of the helium atom:

$$(1s)^2\ ,\ (1s)(2s)\ ,\ (1s)(2p)\ \text{and}\ (1s)(3d)\,.$$

For each of these four configurations, write down the possible values for the following quantities and explain your answers for each example:

(c) the total orbital angular momentum quantum number *L*; [2]

(d) the total spin quantum number *S*; [2]

(e) the total angular momentum *J* for each possible value of *L* and *S*. [2]

END OF QUESTION PAPER