The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring Semester 2006-2007

ELECTROMAGNETISM **2 HOURS**

Answer questions ONE (COMPULSORY) and TWO others.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

TURN OVER

1. COMPULSORY

(a) Two point charges of +15 nC and −8 nC are placed 4 cm apart. Find the electric field and electric potential at the mid-point of a line joining the charges. [1]

(b) An electron is released from rest a distance 150 cm from a stationary point charge $+1\times10^{-9}$ C. Calculate the speed of the electron when it is 30 cm from the point charge. [1]

(c) Three point charges of +6 μC are placed at the coordinates (−0.02, 0), (0, 0) and (+0.02, 0), where distances are measured in metres. Calculate the potential energy of this configuration relative to the configuration where the charges are all an infinite distance apart. [1]

(d) A parallel plate capacitor has plates of area 2×10^{-5} m^2 and separation 0.8 mm. If the charge of the capacitor is 85 nC, calculate the change in the stored energy if the region between the plates is changed from a vacuum to a dielectric of relative permittivity 2.5. [1]

(e) A student determines an electrostatic field to have the form $\mathbf{E} = xy^2\,\hat{\mathbf{i}} - x^2y\,\hat{\mathbf{j}} + xyz\,\hat{\mathbf{k}}$. Is it possible for purely static charges to give a field of this form? [1]

(f) A proton with velocity $2\times10^5\,\hat{\mathbf{i}}$ m s^{-1} enters a region of space where there is a magnetic field given by 1.5 $\hat{\mathbf{k}}$ T. Calculate the initial magnetic force which acts on the proton. [1]

(g) A circular coil of wire of radius 2 cm and containing 3000 turns is placed in a magnetic field, the strength of which is reduced linearly from 2.5 to 0 T in 0.5 s. The normal to the plane of the coil is parallel to the direction of the magnetic field. Calculate the voltage induced in the coil. [1]

(h) Show that the dimensions of the permittivity of free space ε_0 are $C^2\,s^2\,kg^{-1}m^{-3}$. [1]

(i) The speed of light in a material is measured to be 1×10^8 m s^{-1}. If the material has a value of the relative permeability $\mu_r = 1.2$, what is the value of the relative permittivity ε_r ? [1]

(j) A laser beam has a power of 3W and a radius of 1 mm. Calculate the radiation force which would act on a reflecting spherical particle of radius 10 μm when placed in this beam. [1]

CONTINUED

2.

(a) Given Gauss's law in the integral form, $\oint \mathbf{E}\cdot d\mathbf{S} = \sum Q/\varepsilon_0$, derive the differential form $\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$. [1]

(b) A capacitor consists of two concentric hollow metal spheres of radii a and b ($b > a$). Using Gauss's law, derive an equation which gives the **E**-field in the region between the spheres when a charge of $+Q$ is placed on the inner sphere and $-Q$ on the outer sphere. [2]

(i) What is the **E**-field within the inner sphere and outside the outer sphere? Justify your answers. [1]

(ii) Find the potential difference between the spheres and hence the capacitance of the system. [2]

(iii) The region between the spheres is filled with a dielectric of relative permittivity 1.5 and breakdown strength 4×10^5 V m^{-1}. Calculate the capacitance for spheres of radii $a = 3$ and $b = 6$ cm. What is the maximum charge that can be placed on the spheres and what is the maximum energy that the capacitor can store? [2]

(c) A system consists of three concentric hollow metal spheres of radii a, $2a$ and $4a$, carrying charges $+Q$, $+Q$ and $-2Q$ respectively. Derive equations for the **E**-field in the two regions $a \le r \le 2a$ and $2a \le r \le 4a$ and hence show that the magnitude of the potential difference between the inner and outer spheres is

$$|V| = \frac{Q}{4\pi\varepsilon_0 a} \ .$$ [2]

TURN OVER

3.

(a) Show that Ampère's circuital law $\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 \sum I$ leads to the differential equation $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$. [1]

(b) Using Ampère's circuital law, or otherwise, show that the magnetic field a distance r from an infinitely long straight wire carrying a current I is

$$B = \frac{\mu_0 I}{2\pi r}.$$

Sketch the form of the resultant magnetic field showing the relationship between its direction and the direction of the current. [3]

(c) Two infinitely long, parallel straight wires are placed a distance a apart. If each wire carries a current I, use the equation given in (b) to calculate the total magnetic field at the midpoint between the wires for the cases when the directions of the two currents are

(i) identical (parallel) and
(ii) opposite (anti-parallel).

For both cases sketch the variation of the total magnetic field with distance along a line passing through both wires and perpendicular to their axes. [2]

(d) An infinitely long, cylindrical conductor of radius a carries a total current I distributed uniformly across the conductor. Derive expressions for the magnetic field at a distance r from the centre of the conductor for the cases

(i) $r < a$ and
(ii) $r > a$.

Sketch the variation of the field with r. [3]

(e) What is the maximum magnetic field produced by a cylindrical conductor of radius 10 cm carrying a current of 2×10^6 A? [1]

4.

(a) Given Faraday's law of electromagnetic induction, $\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$, derive the equation

$\nabla \times \mathrm{E} = -\frac{\partial \mathrm{B}}{\partial t}$ and show that it is dimensionally correct. [2]

(b) A coil of wire consisting of a single turn of initial radius r_0 is oriented such that the normal to the coil is parallel to a magnetic field B. The coil is imploded such that the radius decreases as a function of time, t, as

$$r = r_0\left(1 - \frac{t}{\tau}\right)$$

where τ is a constant and $0 \leq t \leq \tau$.

Show that the magnitude of the voltage induced in the coil is given by

$$\frac{2B\pi r_0^2}{\tau}\left(1 - \frac{t}{\tau}\right).$$

What is the maximum voltage induced for parameters $B = 2.5$ T, $r_0 = 1$ cm and $\tau = 1$ ms? [3]

(c) Two parallel and horizontal metal rails are placed a distance d apart. A metal bar is free to slide along these rails. If the bar moves with a constant velocity v in the presence of a uniform, vertical magnetic field B, show that the voltage, V, induced between the rails is given by

$$V = Bvd.$$

Draw a diagram showing the relationship between the directions of the velocity, magnetic field and induced current. [2]

(d) The two rails of zero resistance are inclined at an angle θ to the horizontal and a resistor of resistance R is connected between them. If the bar is released from rest, show that when it reaches a velocity v a current

$$I = \frac{Bvd\cos\theta}{R}$$

flows through the resistor and that the bar tends to a terminal velocity given by

$$\frac{mgR\sin\theta}{B^2 d^2 \cos^2\theta},$$

where m is the mass of the bar and g is the acceleration due to gravity.
[Friction may be neglected.] [3]

TURN OVER

5.

(a) The boundary conditions for **E-**, **D-**, **B-** and **H-**fields at the interface between two materials in the absence of any surface charge or conduction currents are:

D and **B** Normal components continuous;
E and **H** Tangential components continuous.

Briefly explain the physical arguments that lead to these conditions. [2]

(b) An electromagnetic wave travelling in a medium of refractive index n_1 is normally incident at the interface with a second material of refractive index n_2. Show that the reflection coefficient r (defined as $r = E_r / E_i$, where E_r and E_i are, respectively, the amplitudes of the reflected and incident **E**-fields) and transmission coefficient t (defined as $t = E_t / E_i$, where E_t is the amplitude of the transmitted **E**-field) are given by

$$r = \frac{n_2 - n_1}{n_2 + n_1} \qquad t = \frac{2n_1}{n_2 + n_1}.$$

(You may assume the **E-** and **H-** components of an electromagnetic wave are related by $H = nE/(\mu_0 c)$ where n is the appropriate refractive index, μ_0 is the permeability of free space and c is the speed of light in a vacuum.) [4]

(c) Show further that the power reflection (R) and transmission coefficients (T) are given by

$$R = \frac{(n_2 - n_1)^2}{(n_2 + n_1)^2} \qquad T = \frac{4n_1 n_2}{(n_2 + n_1)^2}$$

and also that these equations satisfy the conservation of energy.
(You may assume that the power density transmitted by an electromagnetic wave is proportional to nE^2.) [2]

(d) A laser beam is normally incident from a vacuum onto a sheet of transparent material of refractive index 2.5. Calculate the fraction of the energy of the beam that passes through the sheet (neglect multiple internal reflections). [2]

END OF QUESTION PAPER