The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring Semester 2006-2007

SOLIDS **2 HOURS**

Answer question ONE (COMPULSORY) and TWO others.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

TURN OVER

1. COMPULSORY

(a) Lead has a Debye temperature of 100 K. If its molar heat capacity at 10 K is 1.941 J K^{-1} mol^{-1}, what is its molar heat capacity at

(i) 5 K and

(ii) 300 K? [2]

(b) Sketch the phonon dispersion curves for a diatomic solid, labelling each of the four curves as appropriate. Label the abscissae with the maximum wavevector in terms of the interatomic spacing. [2]

(c) The Hall coefficient, R_H is given by $R_H = -1/ne$ where n is the free electron density and e is the electronic charge. It is known that $R_H ne = -0.9$ for potassium. What is the Hall field when a magnetic field of 10 mT is applied to a conductor through which a current density (charge flux) of 1000 C/m^2 s^{-1} flows? The field is applied perpendicular to the current flow. [1]

(d) If $\mathbf{R} = n_1\mathbf{a}_1 + n_2\mathbf{a}_2 + n_3\mathbf{a}_3$ (where n_1, n_2, and n_3 are integers and $\mathbf{a}_1$, $\mathbf{a}_2$, and $\mathbf{a}_3$ are primitive lattice vectors) is a position on a Bravais lattice, show that

$\mathbf{b}_1 = 2\pi \dfrac{\mathbf{a}_2 \times \mathbf{a}_3}{\mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3)}$ is a vector on the reciprocal lattice. [1]

(e) Compare and contrast, with the aid of simple sketches, an x-ray scattering method used to obtain the orientation of a known crystal with a method used to study the structure of a powdered crystalline material. [2]

(f) Sodium chloride is a good example of a face-centred cubic lattice with a basis. Explain what this means, with the aid of a sketch of the crystal structure of this molecule. [1]

(g) Diamond has a Young's modulus of 1 TPa. By what length would a rod-shaped diamond of cross-sectional area 1 mm^2 and length 4 mm extend, if 10^5 N is applied along its axis? [1]

CONTINUED

2.

(a) Describe how the standing wave modes of the Debye model are an improvement on the Einstein model for the thermal behaviour of crystals. [2]

(b) Calculate the molar heat capacity of a solid according to the Einstein model as the temperature tends towards absolute zero. The average thermal energy of a 1-dimensional oscillator is given in the Einstein model by

$$\langle E \rangle = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp\left(\hbar\omega / k_B T\right) - 1}.$$

Here T is the absolute temperature, k_B is Boltzmann's constant and ω is the angular frequency of excitation of the vibrational mode. [3]

(c) The density of states in the Debye model is given by

$$g(\omega) = \frac{V\omega^2}{2\pi^2 v^3},$$

where v is the speed of sound and V is the volume of the sample.
Show that $v = \dfrac{k_B\Theta_D / \hbar}{\sqrt[3]{6\pi^2 N / V}}$, where Θ_D is the Debye temperature. [3]

(d) Lead has a Debye temperature of 100 K, an atomic mass of 0.207 kg/mol and a density of 11340 kg/m^3. What is the theoretically predicted speed of sound in lead? [1]

(e) The ***measured*** speed of sound in lead is 1260 m/s. Supposing that your answer in (d) above corresponds to the longitudinal speed of sound, what would be the transverse speed of sound, assuming that it has only one value in lead? [1]

TURN OVER

3.

(a) Show that the energy of the free electrons in a cube of metal of side L is given by

$$E = \frac{\hbar^2 \pi^2}{2m_e^2 L^2}\left(n_x^2 + n_y^2 + n_z^2\right),$$

where n_x, n_y, and n_z are integers and m_e is the electron mass. [3]

(b) How many electrons can have energy $E = \dfrac{7\hbar^2\pi^2}{m_e^2 L^2}$? [1]

(c) Define the Fermi energy, ε_F. [1]

(d) Given that the density of states for free electrons is given by

$$g(E) = \sqrt{2m_e^3 E}\,\frac{V}{\pi^2\hbar^3},$$

where V is the volume of the metal containing N free electrons, show that

$$\varepsilon_F = \frac{\hbar^2}{2m_e}\left(\frac{3\pi^2 N}{V}\right)^{2/3}.$$ [2]

(e) At low temperatures, the molar heat capacity for sodium is given by

$$C = \gamma T^p + \beta T^m,$$

where $\gamma = 10.9$ J mol^{-1} K$^{-(p+1)}$ and $\beta = 0.49$ mJ mol^{-1} K$^{-(m+1)}$.

Explain how a measurement of the low temperature molar heat capacity of sodium can reveal the separate electronic and thermal (lattice vibration) contributions to C. What are the values of m and p?

Below what temperature will the electronic component dominate the heat capacity? [3]

4.

(a) State one advantage and one disadvantage of using neutrons instead of electrons to deduce the structure of crystals. [1]

(b) Explain the use of a moderator in neutron scattering experiments. [2]

(c) Liquid methane boils at ~112 K at atmospheric pressure. Sketch the neutron intensity as a function of wavelength after exiting a liquid methane moderator at 110 K. Calculate the wavelength at which the neutron flux is the largest and indicate this value on your sketch. [3]

(d) State and prove, with the aid of a sketch, the Bragg condition. [2]

(e) A neutron scattering experiment on a rock salt (NaCl) crystal is performed with monochromatic neutrons of wavelength 0.25 nm. A Bragg peak is observed for $\theta = 26.3^{\circ}$. Given that the lattice parameter of NaCl is 0.564 nm and it has a face centred structure with a basis, identify the Bragg peak and the constitution of the associated atomic planes. [2]

5.

(a) The Poisson's ratio of many materials is between 1/3 and 1/4. Rubber has a Poisson's ratio of 0.5 and the value of cork is slightly negative. What interpretation can be put on these latter values and what physical structures in materials can give rise to a negative Poisson's ratio? Would you expect the Poisson's ratio of steel to be closer to that of cork or rubber, or somewhere in between? [3]

(b) Stainless steel has a shear modulus of 79 GPa. A bar of steel 4 m long with square faces of side 5 cm is subject to shear along the long side. What force needs to be applied in order for one side to be displaced by 0.1 mm? [2]

(c) Describe three different kinds of defects in crystals and indicate which of these defects are mobile. [3]

(d) How does deforming steel by shear contribute to preventing the mobility of defects? Explain why such defects also do not move in alloys. [2]

END OF QUESTION PAPER