DEPARTMENT OF PHYSICS AND ASTRONOMY

Autumn Semester 2006-2007

THERMAL PHYSICS   **2 HOURS**

Answer question ONE (COMPULSORY) and TWO others.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

A formula sheet and table of physical constants is attached to this paper.

1.   COMPULSORY

(i)   Explain what is meant in thermodynamics by
       a) an isothermal process and
       b) an adiabatic process.
       Give one (non-trivial) physical example of each.   [1]

(ii)   State the First Law of Thermodynamics, explaining the significance of the signs
       of the various terms.   [1]

(iii) A sample comprising 7 moles of a certain ideal gas has a heat capacity at constant
       volume of 87.26 J K^{-1}. Calculate the heat capacity at constant pressure.   [1]

(iv)   An ice/water mixture in thermal equilibrium at atmospheric pressure is used as the
       cold reservoir of a Carnot engine. What must be the temperature of the hot reservoir
       if the engine is to be 80% efficient?   [2]

(v)   A can of beer, at an initial temperature of 4°C, and of heat capacity 2500 J K^{-1}, is
       allowed to reach thermal equilibrium with its surroundings at 20°C. Making
       appropriate assumptions, calculate the entropy change of the universe as a result
       of this process.   [3]

(vi)   In the hydrogen atom, the lowest electron energy level E_0 is separated from the first
       excited level E_1 by an energy of 10.2 eV. Assuming that E_0 is non-degenerate, while
       E_1 is 4-fold degenerate, calculate the probability of the electron occupying E_1 relative
       that of occupying E_0 at a temperature of 15,000 K.   [2]

CONTINUED

2.

(i)   Give a brief explanation of the concept of *equipartition of energy*   [2]

(ii)   Using the results of equipartition theory, write down, with justification, expressions
        for the internal energy and heat capacity at constant volume of n moles of
        a) a monatomic gas and
        b) a diatomic ideal gas.   [2]

(iii)   Describe, with the aid of a sketch graph, an experimental observation which shows
        that classical equipartition theory is only of limited validity, and how better
        agreement with experiment is obtained by the adoption of quantum theory.
        Comment on any regimes where the predictions of equipartition theory and quantum
        theory coincide.   [4]

(iv)   Calculate the average vibrational energy of a quantised harmonic oscillator of
        frequency $\nu = 10^{14}$ Hz at temperatures close to absolute zero.   [2]

3.

(i)   Write a brief (~300 word) account of the role of entropy in thermodynamics and
        statistical physics. Your account should discuss the macroscopic and microscopic
        interpretations of entropy, the generalised form of the Second Law of
        Thermodynamics, reversible and irreversible processes, and calculations of entropy
        changes.   [5]

(ii)   Calculate the entropy change of 2 moles of an ideal gas when they undergo a
        reversible isothermal compression to one third of the original volume.
        What must the entropy change of the surroundings be as a result of this process?   [2]

(iii)   A box containing 60 coins is shaken up so that n coins have heads up, the remainder
        being tails up.

        a) what is the most probable macrostate of the assembly of coins?   [1]

        b) What is probability of the $n = 25$ macrostate occurring, relative to that of the most
            probable macrostate?   [1]

        c) What is the entropy of the $n = 25$ macrostate?   [1]

TURN OVER

4.

(i)   Explain qualitatively how you would expect the temperature of
       (a) an ideal gas and
       (b) a Van der Waals gas
       to change when the gas undergoes a Joule expansion.   [1]

(ii)   Show that the incremental temperature change for any gas undergoing a Joule
       expansion is given by

$$dT = -\frac{1}{C_V}\left(\frac{\partial U}{\partial V}\right)_T dV$$

       where the symbols have their usual meanings.   [3]

(iii) Show, by using the central equation $dU = TdS - pdV$ and an appropriate Maxwell
       relation that the expression in (ii) may also be written as

$$dT = \left(\frac{1}{C_V}\left(p - T\left(\frac{\partial P}{\partial T}\right)_V\right)\right)dV$$

       Why is this form of the expression more useful?   [3]

(iv)   Calculate the change in temperature when 1 mole of nitrogen gas undergoes a Joule
       expansion from a volume of 0.1 m^3 to a volume of 0.2 m^3 (assume the gas obeys the
       Van der Waals equation of state).   [3]

You may find the following information useful:

Van der Waals equation of state is $\left(P + \frac{an^2}{V^2}\right)(V - nb) = nRT$

where for one mole of nitrogen gas,
$a = 0.15$ J m^3, $b = 4 \times 10^{-5}$ m^3, $C_v = 21$ JK^{-1}
(assume these values to be independent of *P, V, T*):

and the Maxwell's relation, $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$.

CONTINUED

5.

(i) Give a brief outline of the roles of the Gibbs and Helmholtz functions in thermodynamics. [2]

(ii) What can be said about the specific Gibbs functions of two or more phases of a substance, when these phases co-exist in equilibrium? [1]

(iii) Starting from the Clausius-Clapeyron equation,

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L}{T\Delta v},$$

and using appropriate assumptions, show that for a liquid $\rightarrow$ vapour phase transition, the transition temperature is related to the pressure by

$$T = \frac{L}{R\ln\left(\dfrac{P_0}{P}\right)},$$

where P_0 is a constant. [4]

(iv) In low temperature experiments, temperatures below that of the normal boiling point of liquid helium (4.2 K) may be achieved by reducing the pressure above the liquid to decrease the boiling temperature. Calculate the pressure required to reduce the boiling temperature of one mole of helium to 3 K. [3]

The Molar latent heat of vaporisation for liquid helium is $L = 80$ J mol^{-1}.

END OF QUESTION PAPER