The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring Semester 2006-2007

INTRODUCTION TO ASTROPHYISCS **2 HOURS**

Answer ALL questions in section A and TWO questions from section B.

A formula sheet and table of physical constants is attached to this paper.

The breakdown on the right-hand side of the paper is meant as a guide to the marks that
can be obtained from each part.

SECTION A

Answer ALL questions

1. A star has an effective photospheric temperature of 8000 K, a radius of 8×10^6 m, and a measured parallax of 0.05 arcseconds. Estimate:

(a) the bolometric luminosity of the star;

(b) the bolometric flux from the star at Earth. [5]

2. Four stars (I, II, III, IV) have the following spectral features.

Star I: Strong molecular lines.

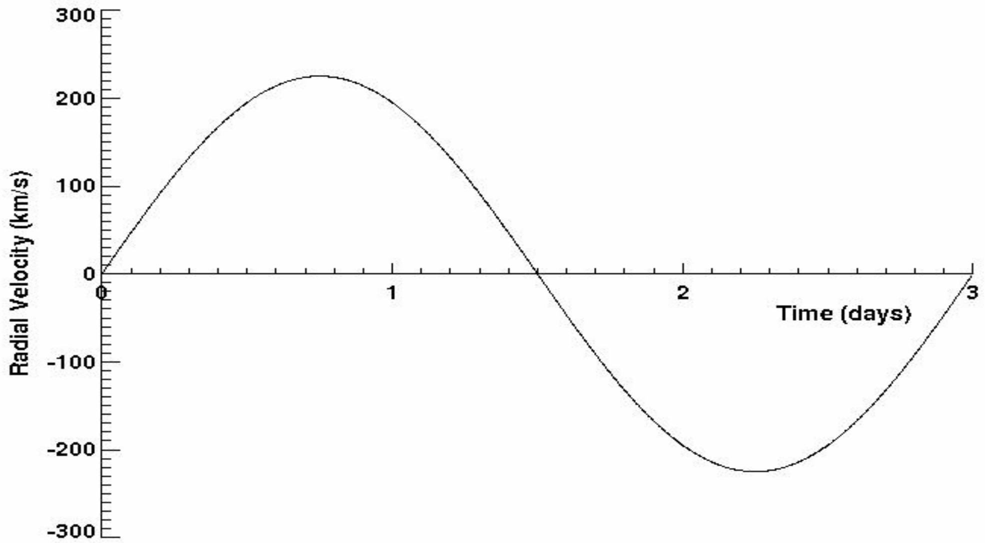
Star II: Strong H lines, weak He lines, weak metal lines.

Star III: Strong He lines.

Star IV: Strong lines of singly ionized calcium.

Explaining your reasoning, place the stars in order of temperature, indicating which is the hottest and which is the coolest. If star I was photographed on colour film, would its colour appear to be red or blue? [5]

3. Figure 1 displays the radial velocity curve of the secondary star (object A) in a single-lined spectroscopic binary system.



Given that

$$\frac{\left(v_A^{obs}\right)^3 P}{2\pi G} = \frac{m_B^3}{\left(m_A + m_B\right)^2}\sin^3 i$$

(where v_A^{obs} is the observed radial velocity of the secondary star, P is the orbital period of the secondary star, m_A and m_B are the masses of the secondary and primary stars respectively, and i is the angle of inclination between the plane of the sky and the orbital plane of the binary system):

(i) estimate a lower limit on the mass of the primary star (object B). [4]

(ii) What type of astrophysical object is the primary? [1]

4. Derive the equation of hydrostatic equilibrium for an atmosphere:

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{GM(r)\rho(r)}{r^2}$$

where $P(r)$ and $\rho(r)$ are the pressure and mass density at radius r, $M(r)$ is the mass interior to radius r, and G is the gravitational constant.

Give two examples of practical situations in which this equation applies. [5]

SECTION B

Answer TWO questions

5. Describe the conditions under which thermal (black body) continuum radiation is emitted. Give two examples from astronomy of sources of black body radiation, and describe briefly one other continuum emission mechanism. [5]

The monochromatic flux emitted by the surface of a black body is given by the following equation (in frequency units):

$$F_{\nu}(T) = \frac{2h\pi\nu^3}{c^2} \frac{1}{\exp\left(h\nu/kT\right)-1} \quad \left(\mathrm{W\ Hz^{-1}m^{-2}}\right).$$

Show that the monochromatic flux at the surface of a black body in wavelength units is given by

$$F_{\lambda}(T) = \frac{2h\pi c^2}{\lambda^5} \frac{1}{\exp\left(hc/\lambda kT\right)-1} \quad \left(\mathrm{W\ nm^{-1}m^{-2}}\right).$$ [5]

It can be shown that the bolometric *surface* flux of a star is related to its photospheric temperature by the equation

$$F_{bol}\left(T\right) = \sigma T^4.$$

Derive an expression for the observed bolometric flux (F_{bol}) of a star at a distance d, in terms of its temperature (T) and photospheric radius (R). [2]

The bolometric fluxes and spectral energy distributions of two stars (X and Y) in a distant binary star system have been measured from Earth. The spectral energy distribution of star X peaks at 300 nm, while that of star Y peaks at 600 nm.
If star X has a bolometric flux 100 times larger than that of star Y, what is the ratio, R_X/R_Y, of the radii of stars X and Y? [3]

6. Show that the energies of the discrete energy levels in hydrogen atoms are given by

$$E_n = \frac{-A}{n^2}$$

where $n = 1, 2, 3...$ and A is a constant. [5]

Sketch the energy levels given by this formula and indicate the *ground state*. Use the diagram to show the meaning of the term *ionization energy*. [3]

If the value of A is 2.178×10^{-18} J, what is the wavelength of the spectral line which arises from a transition between the *first and second excited states* of neutral hydrogen? [2]

An absorption line in this series (i.e. in the series which has a principal quantum number of $n = 2$) is observed at a wavelength of 486.1 nm. Mark on your diagram the transition which produces this absorption line, and calculate the wavelength at which this series converges to a limit. [3]

The Balmer absorption lines of hydrogen vary in strength along the Harvard Spectral Classification sequence. Sketch the trends observed in line strengths with spectral type, and briefly explain the observed trends in terms of physical processes. [2]

7. Write detailed accounts of any *three* of the following:

(i) Continuum, emission line and absorption line spectra and the Kirchoff-Bunsen laws, including appropriate astronomical examples.

(ii) The evidence for the existence of Black Holes.

(iii) The Doppler effect and its applications in astrophysics.

(iv) The different ways in which stellar masses can be measured. [15]

TURN OVER

8. PLEASE SEE SEPARATE SHEET

(a) A spiral galaxy lies at a distance of 100 Mpc, and optical imaging of its starlight shows that it has a maximum angular diameter of 60 arcseconds. If all stars in this galaxy are on circular orbits, calculate the maximum orbital radius, in kpc, for a star in this galaxy. [2]

(b) *Rotation curves* can be constructed for spiral galaxies, and used to estimate the mass of the galaxy within a particular radius. Explain what sort of observations are needed to make a rotation curve, and how measurements are made extending well beyond the optical (stellar) emission of the galaxy. Derive an expression for the mass of a galaxy within a radius r in terms of the observed velocities, assuming that the plane of the galaxy disk is inclined to the plane of the sky by an angle i. List any assumptions made. [6]

(c) Figure 2 (**see separate sheet**) displays the rotation curve for the galaxy.

(i) If the plane of the galaxy disk is inclined to the plane of the sky by an angle $i = 60^\circ$, calculate the total mass of the galaxy within $r = 40$ kpc. [1]

(ii) Explain how $M(r)$ (the mass within a radius r) depends on radius between

(1) $0 < r < R$, where $R = 10$ kpc, and

(2) $r > R$.

Sketch a graph of $M(r)$ as a function of radius, out to the maximum radius at which the radial velocity is measured. [3]

(iii) The bulk of the visible starlight lies within a radius of $r = R$. If ALL the mass of the galaxy lies within a radius of $r = R$, how would you expect $v(r)$ to depend on r at radii $r > R$? Sketch this on Figure 2, and explain what this implies about the matter content of spiral galaxy haloes. [3]

END OF QUESTION PAPER

PHY104

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QUESTION 8

Remember to attach this annotated figure to your answer sheets.

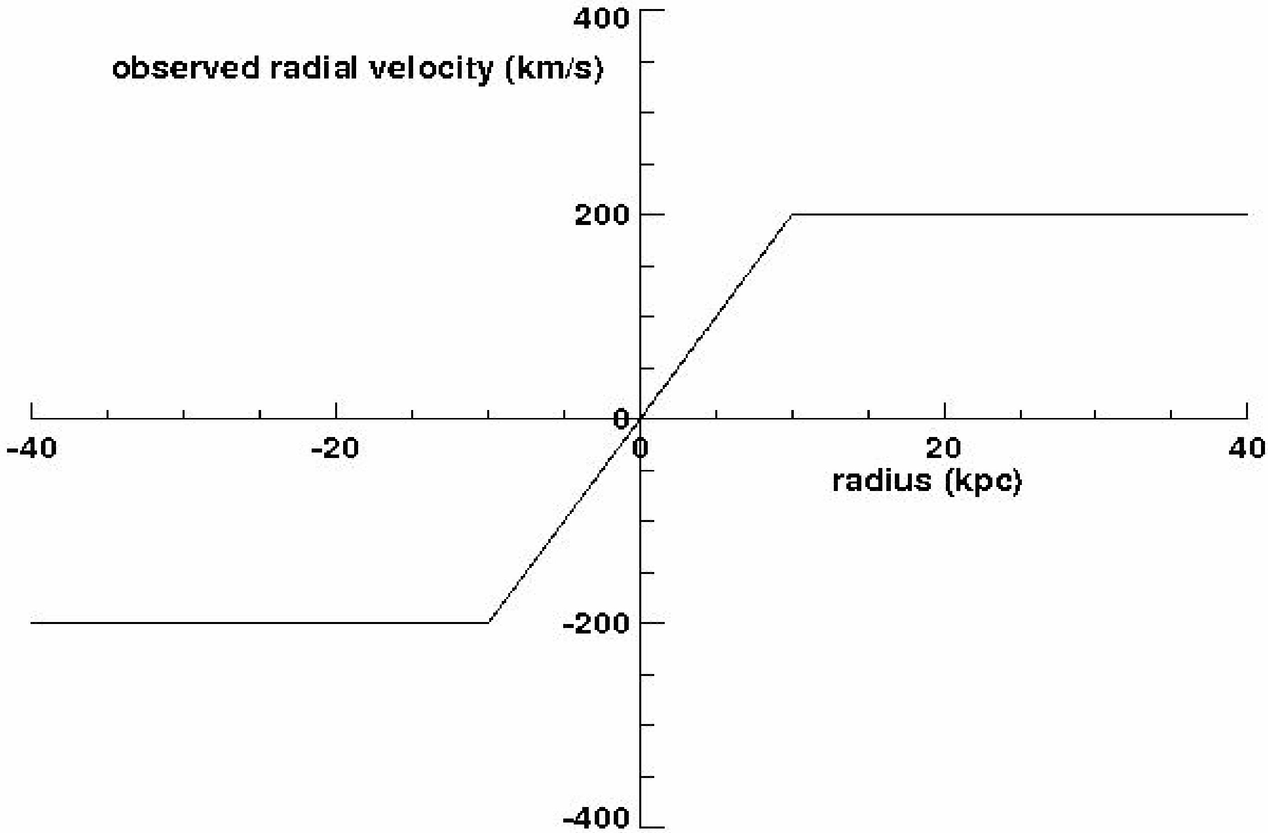

Figure 2: (Question 8) the rotation curve for the galaxy.

Remember to attach this annotated figure to your answer sheets.