The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Spring Semester 2006-2007

FIELDS, WAVES AND QUANTA **3 HOURS**

Answer questions ONE and SIX (COMPULSORY) and FOUR others, including at least one from each section.

Answers to different sections must be written in separate books, the books tied together and handed in as one.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

TURN OVER

SECTION A

1. COMPULSORY

(a) Steel has a resistivity of 1.8×10^{-7} Ω m. What will be the resistance of a wire 10 m long and 0.5 mm in diameter? [1]

(b) Two resistors R_1 and R_2 are connected in series, and their combined resistance is measured as 18 Ω. They are then connected in parallel, and their new combined resistance is 4 Ω. What are the two individual resistances? [2]

(c) Use Ampère's Law to determine the magnetic field a distance r from a long straight wire carrying a current I. [1]

(d) State Kirchhoff's circuit rules. [2]

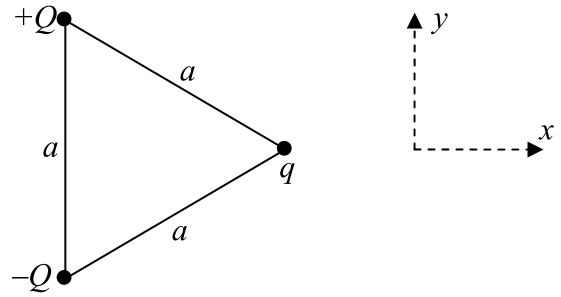
(e) A current of 3 A is passed though a rectangular coil consisting of three turns 5 cm by 8 cm. What is its magnetic dipole moment? The plane of the coil is parallel to a magnetic field of 0.1 T. Calculate the torque acting on the coil and its magnetic potential energy. [2]

(f) A battery has a terminal voltage of 11.75 V when it is supplying 5 A and 8.75 V when it is supplying 25 A. Calculate its EMF and its internal resistance. [2]

2.

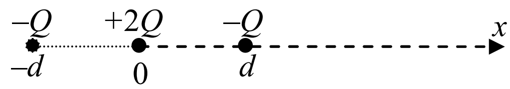
(a) What is Coulomb's Law for the force between two charges? Give the vector equation for the force, defining all symbols that you use. [2]

(b) Three charges $+Q$, $-Q$ and q are situated at the vertices of an equilateral triangle of side a as shown. Find the magnitude and direction of the force acting on q.



[3]

(c) A linear quadrupole consists of a charge $+2Q$ at the origin and two charges $-Q$ at $x = \pm d$.



(i) Write down the magnitude of the electric field at a point on the x-axis where $x > d$. [1]

(ii) Use the binomial expansion for the case $x \gg d$ to determine how the field at a point on the x-axis depends on x, in the large distance limit. [3]

(iii) If $Q = 1$ μC and $d = 0.1$ mm, what is the field at $x = 15$ cm? [1]

3.

(a) State in words Gauss's Law in electrostatics. Present this as an equation, defining all symbols that you use. [2]

(b) A small uniformly charged insulating cube carries a total charge of 3×10^{-8} C. It is suspended inside an uncharged metal sphere. What is the total electric flux emerging from the sphere? [1]

(c) A long insulating cylindrical rod of radius a contains a uniform charge density ρ. Use Gauss's Law to determine the electric field at a distance r from the centre of the rod

(i) when $r < a$;
(ii) when $r > a$. [4]

(d) A large flat insulating sheet is suspended away from other charged or conducting objects. The electric field is measured a distance of 50 cm from the sheet, and found to be 150 V m^{-1}, directed towards the sheet. What is the charge density on the sheet, and what magnitude of electric field would you expect 1 cm from its surface? [3]

4.

(a) A parallel-plate capacitor is formed from two metal sheets, each 50 cm by 80 cm, with a gap of 0.5 mm. A potential difference of 150 V is applied across the plates.

(i) What is the electric field between the plates? [1]
(ii) What will be the electric charge on each plate? [1]
(iii) What is the capacitance? [1]

(b) How would the capacitance be modified if the gap was filled by an insulating material with a dielectric constant of 5? Explain what happens to the dielectric on the microscopic scale when placed in an electric field. [3]

(c) A 10,000 μF capacitor is connected to a potential of 5 V. Subsequently, the capacitor is disconnected from the source of potential, and a 5 kΩ resistor is connected across it. By considering the current flowing through the resistor as a function of time *t*, demonstrate that the charge on the capacitor is given by

$$Q = Q_0 e^{-t/RC}$$

where Q_0 is the initial charge, *R* is the resistance and *C* is the capacitance.
How long will it take for the voltage across the capacitor to fall to 2 V? [4]

5.

(a) Write down the vector expression for the force on a moving charged particle, defining all the symbols that you use. In which direction does the force act if the field is

(i) parallel to the motion;
(ii) perpendicular to the motion? [2]

(b) An electron is travelling along the *x*-axis with a velocity $2 \times 10^6 \text{ m s}^{-1}$. It enters a region of uniform magnetic field of strength 0.2×10^{-3} T parallel to the *z*-axis. Describe its subsequent motion as precisely as you can. How long does it take to return to its original position? [4]

(c) Use the Biot-Savart Law to determine the magnetic field at a point on the axis of a circular coil. If it consists of 50 turns of wire of radius 5 cm, carrying a current of 2 A, what is the field at the centre of the coil? How far away from the centre (along the axis) has the field fallen to half this maximum value? [4]

SECTION B

6. COMPULSORY

(a) A particle undergoes simple harmonic motion with amplitude 10 cm and frequency 5 Hz. If it passes through the equilibrium position ($x = 0$) at time $t = 0$, calculate the position, velocity and acceleration of the particle at $t = 3.5$ ms. [2]

(b) A 2 kg mass is suspended from a steel wire of diameter 1 mm and length 0.75 m. Calculate the speed of transverse waves along the wire. [1]

(c) A car is approaching a stationary pedestrian at a speed of 15 m s^{-1}. If the driver of the car blows the horn (frequency 300 Hz) what frequency does the pedestrian hear? [1]

(d) In a Young's double slit experiment, light of wavelength 630 nm is incident on a pair of slits separated by 1.0 μm, and the resulting interference pattern is observed on a screen 30 cm behind the slits. Calculate the distance on the screen between successive maxima in the interference pattern. [1]

(e) Explain briefly what is meant by the *work function* of a metal, and how it might be measured experimentally. [1.5]

(f) Calculate the de Broglie wavelength of an electron accelerated through a potential difference of 4.5 kV. [1]

(g) Sketch the wave functions of the three lowest energy levels for a particle confined in an infinite potential well of width L. [1]

(h) If the particle in the previous question is a proton, and $L = 15$ nm, calculate the energy difference between the ground state and the first excited state. [1.5]

TURN OVER

7.

(a) Explain what is meant by *longitudinal* and *transverse* waves. Give one example of each type of wave motion. [2]

(b) Write down an expression for the displacement as a function of position and time of a string carrying a harmonic transverse wave of amplitude A, angular frequency ω and wavenumber k, propagating in the negative x-direction. [1]

(c) Show that the average power carried by the wave in part (b) is given by

$$P_{ave} = \frac{1}{2}\sqrt{T\mu}\,\omega^2 A^2$$

where T is the tension in the string and μ is its mass per unit length. [3]

(d) A piano wire of mass 4 g and length 90 cm is stretched with a tension of 30 N. A wave of frequency 440 Hz and amplitude 2 mm travels along the wire. Calculate:

(i) the average power carried by the wave; [1]
(ii) the sound intensity at a distance of 3 m from the wire; [1.5]
(iii) the sound intensity at a distance of 12 m from the wire relative to that at 3 m. Express your answer in dB. [1.5]

8.

(a) Briefly describe the *phasor method* for finding the combined effect of two or more oscillations of the same frequency but with a constant phase difference between them. Illustrate your answer with an appropriate diagram. [2]

(b) Give a brief description of Young's double slit experiment, including experimental setup and expected results. [2]

(c) Show that the phasor sum of two oscillations

$$E_1 = E_0 \cos(\omega t)$$
$$E_2 = E_0 \cos(\omega t + \phi)$$

is given by

$$E_{TOT} = 2E_0 \cos\left(\frac{\phi}{2}\right).$$

If E_0 is the amplitude of the electric field of the incident light waves in a Young's double slit experiment, explain why E_{TOT}^2 is the quantity of interest for determining the intensity distribution of the interference pattern. [3]

(d) From the results of part (c) show that the intensity distribution in the double slit experiment is given by

$$I_{TOT} = 4I_0 \cos^2\left(\frac{\pi d \sin\theta}{\lambda}\right)$$

and explain the meaning of variables I_0, d, θ and λ. [3]

CONTINUED

9.

(a) Explain what is meant by the *photoelectric effect* and the *Compton effect*, and briefly describe (with the aid of diagrams) how each of these effects may be demonstrated experimentally. [4]

(b) How do the results of the above experiments support the photon model of electromagnetic radiation? [3]

(c) Describe the Rutherford scattering experiment and explain what the results of this experiment reveal about the structure of the atom. [3]

END OF QUESTION PAPER