The University Of Sheffield.

DEPARTMENT OF PHYSICS AND ASTRONOMY

Autumn 2006-2007

MECHANICS HEAT AND MATTER **3 HOURS**

Answer questions ONE and FIVE (COMPULSORY) and THREE others, including at least one from each section.

Answers to different sections must be written in separate books, the books tied together and handed in as one.

A formula sheet and table of physical constants is attached to this paper.

All questions are marked out of ten. The breakdown on the right-hand side of the paper is meant as a guide to the marks that can be obtained from each part.

TURN OVER

SECTION A

1. COMPULSORY

a) Write down the relationship between pressure, volume and temperature for an ideal gas, defining any symbols used. [2]

b) How much energy is required to melt one tonne (1000 kg) of scrap lead, initially at room temperature (20 °C)? [2]

c) Explain why incandescent light bulbs are not a very efficient way of converting electrical energy into visible light, and give two examples of more efficient light sources. [2]

d) What limitations does the second law of thermodynamics place on the efficiency with which heat energy can be converted into mechanical work? [2]

e) Calculate the root mean squared velocity of oxygen molecules in air at room temperature (20 °C). [2]

[For lead: specific heat capacity (which can be assumed to be constant over this temperature range) 128 J kg^{-1} K^{-1}, latent heat of fusion 24,500 J kg^{-1}, melting point 327 °C. The mass of an O_2 molecule $m(O_2)$ = 32 u.]

CONTINUED

2.

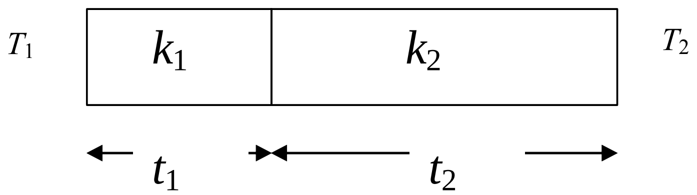
a) The thermal conductivity, k, of a material is defined through the equation

$$\frac{\Delta Q}{\Delta t} = -kA\frac{\Delta T}{\Delta x}.$$

With the aid of a diagram, define the other terms in this equation. [1]

b) A stainless steel saucepan has a base with an area 0.15 m^2 and thickness 8.5 mm. The pan is full of boiling water, to which heat is being supplied at the rate of 5000 W. Calculate the temperature of the bottom of the pan, assuming that the cooker uniformly heats the whole area of the pan's bottom. [2]

c) Two bars of different materials, with thermal conductivities k_1 and k_2 are joined together as sketched in the diagram. Derive an expression for the heat flow through the composite bar, when its ends are maintained at temperatures T_1 and T_2. [3]



d) A more expensive saucepan also has a base area of 0.15 m^2, but its base consists of 6 mm of copper joined to a lining of 2.5 mm of stainless steel. Calculate the base temperature when 5000 W is being supplied to boiling water in the pan. [2]

e) Why do professional cooks tend not to use solid stainless steel pans? [2]

[Thermal conductivity of stainless steel: 8.0 W m^{-1} K^{-1}, copper: 385.0 W m^{-1} K^{-1}.]

3.

a) Show that the ideal gas law can be rewritten in the form

$$P = nk_B T,$$

where n is the number density of the gas, i.e. the number of molecules per unit volume. [2]

b) Consider a model of the atmosphere, in which the temperature is everywhere constant, but in which the pressure $P(h)$ decreases with increasing height h.

(i) By considering a slice of gas of thickness dh at height h in a cylindrical column of gas of area h, write down an expression for dP, the difference in pressure between the top and bottom of the slice. [2]

(ii) Using the result of (a), rewrite your expression to obtain a differential equation relating the pressure P and the height h.

Show that the solution of this equation is

$$P(h) = P_{h=0} \exp(-mgh/k_B T),$$

where m is the mass of a gas molecule. [2]

c) Give a physical interpretation of this result in terms of the Boltzmann distribution. [2]

d) Taking the average mass of an air molecule to be 4.78×10^{-26} kg, estimate the height of a mountain at whose summit the air pressure would be half the value at sea level. [2]

4.

a) State the entropy form of the second law of thermodynamics. [1]

b) Write down an expression for the change in entropy ΔS of a system when heat Q is added at constant temperature T. How must this expression be modified to account for situations in which the temperature is not constant? [3]

c) 1 kg of water is reversibly cooled from 100 °C to 50 °C.

(i) What is the change of entropy during this process? [1]
(ii) What is the total change of entropy of the universe during this process? [1]

d) 1 kg of water at 100 °C is added to 1 kg of water at 0 °C in an insulated container.

(i) What is the net change in entropy of the water? [2]
(ii) What is the change in entropy of the universe during this process? Why is it not zero? [2]

[The specific heat capacity of water, which you can assume is constant over this temperature range, is 4190 J kg^{-1} K^{-1}.]

SECTION B

5. COMPULSORY

(a) A rifle is aimed horizontally at a target 50 m away. The bullet strikes the target 2.0 cm below the aim point. What was the bullet's flight time? What was the bullet's speed as it left the barrel? [2]

(b) A girl drops a coin from rest into a well and measures the time that passes before she hears the sound of the coin splashing into the water to be 3.6 s. Calculate the depth of the well assuming the speed of sound to be 330 m s^{-1}. [4]

(c) A stone of mass 1.2 kg is tied to a piece of fishing line of length 1.3 m which has a breaking strength of 125 N. The stone is whirled around in a **vertical** circular path. Calculate the maximum velocity at which the stone can travel without the line breaking. State also whether you would expect the line to break at the top or bottom of the circular path. [4]

6.

A neutron in a reactor makes an elastic head-on collision with the nucleus of a carbon atom initially at rest.

(i) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? [6]

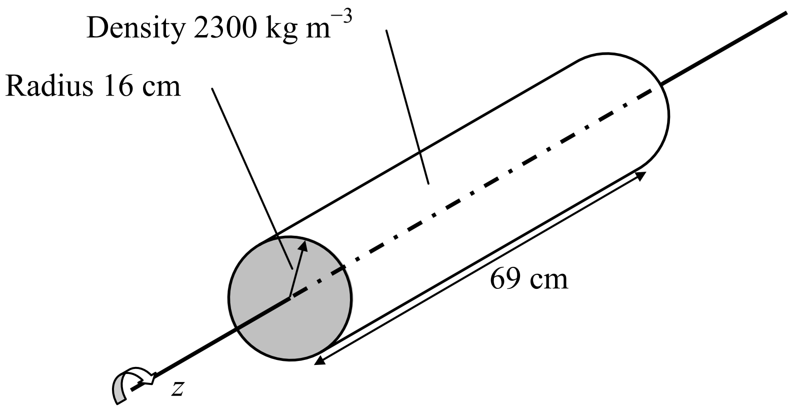
(ii) If the initial kinetic energy of the neutron is 1.6×10^{-13} J, find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision. [4]

[The mass of the carbon nucleus is approximately 12.0 times greater than the neutron mass.]

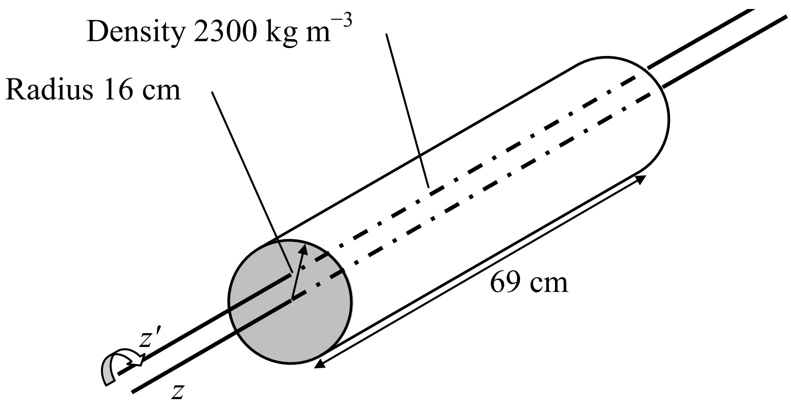
7.

(a) Define the moment of inertia both in words and by providing a simple mathematical definition. [2]

(b) Calculate the moment of inertia of the solid right circular cylinder shown below around its long, central axis (z). [4]



(c) What is the moment of inertia for the cylinder around the z' axis which is parallel to the z-axis and passes through a point 6.0 cm from the z-axis (as shown below)? [2]

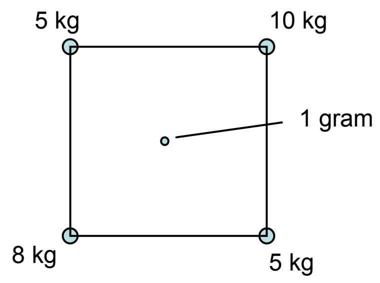


(d) Find the angular momentum of the cylinder in (c) above when it rotates about the z' axis at 28 rpm. [2]

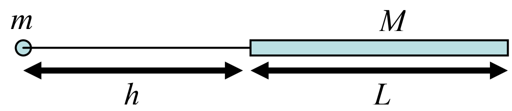
8.

(a) Calculate the acceleration due to gravity at a height of 1000 km above the Earth's surface. (Mass of Earth = 5.98×10^{24} kg, radius of Earth = 6.37×10^{6} m.) [2]

(b) Four point masses are placed at the points (0.00, 0.00) m, (0.00, 3.00) m, (3.00, 3.00) m and (0.00, 3.00) m as shown below. Calculate the resultant gravitational force on a 1 gram mass placed at the centre of the square, assuming that the system of masses is isolated from the rest of the Universe. Give your answer in full vectorial form. [4]



(c) A particle (point mass) of mass m is positioned a distance h from a rigid, uniform bar of length L and mass M, as indicated below.



Show that the force exerted on the particle is given by

$$\mathbf{F}_{\mathbf{g}} = -\frac{GmM}{h(h+L)}\hat{\mathbf{i}}$$ [3]

(d) Two galaxies of dimension L are separated by h where $h >> L$. Using the result from part (c) above, find the inter-galactic gravitational force and state what can be deduced about the behaviour of the galaxies. [1]

9.

(a) Show that the following functions are valid solutions of the general equation for Simple Harmonic Motion (SHM), $\frac{d^2x}{dt^2} = -\omega^2 x$:

(i) $x = A\sin(\omega t + \phi)$; [1]

(ii) $x = Ae^{i\omega t} + Be^{-i\omega t}$. [2]

(b) For a particle undergoing SHM whose displacement is given by $x = A\cos(\omega t)$, show that its total energy at any time t is given by

$$E = \frac{1}{2}kA^2 \quad \text{where } \omega = \sqrt{\frac{k}{m}}.$$ [4]

(c) What is the length of a simple pendulum used on the Moon whose period on the Moon is equal to the period of a similar pendulum of length 2.0 m on Earth?
[Use $g_{\text{Earth}} = 9.82$ m s^{-2} and $g_{\text{Moon}} = 1.64$ m s^{-2}.] [3]

END OF QUESTION PAPER