Chap6: Forecasting Demand in a SC

- Role of Forecasting in a SC
- The 2 extreme cases:

Case 1 (estimate a smaller amount than required- under-estimate)

→loss in sale then revenue

→impact on customer service and reputation

Case 2 (over-estimate)

→ large costly inventory →unsold product →loss in revenue -→waste (environmental and economic)

- Pull process (require response from manager)
- Push process (need to plan the level of activity)

→ Both need a good estimate of the demand

- **Dell Example**: Order PC components using estimate of customer order but do the assembly using the right amount. Forecasting is for # components (push process) and determine the capacity of the plants (pull process).
- Student: choose an example where forecasting is needed.

Characteristics of forecasts

1. Need for expected values of forecast & a measure of forecast error

   For instance expect a demand in the range [100, 200] or [130, 170], both the same average of 150 but the second range is more safer

   → Need for measuring the errors → require good forecasting methods

2. Long term forecast: usually less accurate than short term

3. Aggregate demand: usually more accurate (inflation about 2%, but individual products vary differently)

4. Farther up the SC (farther from consumer), the larger is the error (information gets distorted → bullwhip effect)

Components of a forecast & forecasting methods

Awareness of the company

- **Past demand**
- **Lead time of product**
- **Planned advertising & marketing effort incl planned price discounts**
- **State of the economy**
- **Competitors action (taken or might take)**

Classification of forecasting methods

- **Qualitative (subjective & rely on human judgment, appropriate when little infos is available, new launch of product, inside infos about other markets, etc)**
- **Time series (use historical demand, assume that past is a guide for the future, appropriate when demand pattern is similar, simple to use)**
- **Causal (effect from he environment, find correlation between environmental factors & demand and then use the correlation to estimate the forecast demand)**
- **Simulation (imitate human behaviour & consumer choices to yield a forecast; can combine time series with causal to evaluate “what if scenarios”**

Steps affecting the forecast in SC

- **Understand the objective of forecasting**
  - **Identify the decisions that will be affected by the forecast:**
  - **Example: If Tesco plans price discount for detergent in July → these infos must be known to manufacturer, distributors, etc involved in making and transporting the product → need for a common demand forecast**

- **Integrate demand planning & forecasting in a SC**
  - **Need for a cross functional team →guarantee that the forecast is shared between the activities → appropriate to have a member from each activity involved**

- **Understand/Identify customer segment**
  - **Different methods may be used for each segment (demand value, order frequency, similarities, seasonality, etc)**
  - **Knowing the segment can make the choice of the method more simpler.**

Steps affecting the forecast in SC (cont)

- **Identify major factors that can affect the forecast**
  - **Demand patterns (up, down, steady, seasonal, etc)**
  - **Estimate based on demand not sales** (unmet demand, effect of discount, level of competition, etc)
- **Determine the appropriate forecasting methods**
  - **Understand the 3 dimensions (geography, product group, customer group)**
  - **Choose the technique that fit the dimension, or a combination of methods)**
- **Establish performance & error measures for the forecast**
  - **measure the error and take action to modify the methods if needed)**

Time Series-based Techniques

- Most appropriate when future demand is linked to past demand
- Incorporate random elements
- Observed Demand (O)

O = Systematic Component (S) + Random error

S: expected demand (forecast) made up of:

- current diseasonalised demand

- Trend (rate of growth or decline)

- Seasonality (prediction for seasonal fluctuation)

Random error: forecast error →noise → assess better the accuracy of S

→ Should be centred around zero with a small variation

Time Series-based Techniques (cont)

- Aim to predict (S) & estimate (R)
- Methods can be static (once done, remain unchanged through the periods) or adaptive (continuous updates through the periods)

- Systematic Component (S) can be used as follows:
  - Multiplicative: S = Level x Trend x Seasonal factors
  - Additive: S = Level +Trend +Seasonal factors
  - **Mixed: S = (Level + Trend) x Seasonal factors**

Static Forecasting Methods

- Assume the estimates for Level (L), Trends (T), and Seasonal factors (St) to remain fixed for all months say.
- **Idea: estimate first L, T and S_t using past demand and use the following formula for Forecast**

$$F_{t+l} = [L + (t+l)T]S_{t+l} \qquad (6.1)$$

where F_{t+l} forecast made at time t for the demand in period $(t+l)$ and L,T and S are fixed before hand.

How to Estimate L,T and S

- Deseasonalise the demand to obtain $\overline{D_t}$
- Use Linear Regression of the form $\overline{D_t} = L + t.T$

  to estimate L and T (where L and T represent the intercept and the gradient respectively).
- Compute the new values of $\overline{D_t}$ for all t
- Estimate seasonal factors as: $\overline{S_t} = D_t / \overline{D_t}$
- Compute Mean Seasonal Factor in each cycle (quarter)
- Compute the forecasts F_{t+l} using Equation (6.1)

Calculation (cont)

- Computation of $\overline{D_t}$

$$\overline{D_t} = \begin{cases} [D_{(t-p/2)} + D_{(t+p/2)} + \sum_{i=t+1-p/2}^{i=t-1+p/2} 2D_i] / 2p & p \quad even \\ \sum_{i=t-p/2}^{i=t+p/2} D_i \Big/ p & p \quad odd \end{cases}$$

Where p is the periodicity (# quarters in a year, p=4)

- Mean seasonality factor **(a given quarter):** $S_{t+l} = \frac{\sum_{j=0}^{j=r-1} S_{jp+l}}{r}$

with r: # times a quarter is used in the past

(i.e.; if there are 3 yrs & p=4 quarters per yr → r=3);

Example: $S_{13} = \frac{S_1 + S_5 + S_9}{3}$ (1st quarter) ;.. .; $S_{16} = \frac{S_4 + S_8 + S_{12}}{3}$ (4th quarter)

Case study (Tahoe Salt, p205→)- Static

- The demands (in tonnes of salt produced) of 3 years with 12 periods (4 quarters each). Here p=4 and r=3 observations in each quarter.
- Aim to forecast the next 4 quarters.

Year	Quarter r	Period t	Demand, Dt	Deseasonalised Demand
1	2	1	8,000	
1	3	2	13,000	
1	4	3	23,000	**19,750**
2	1	4	34,000	**20,625**
2	2	5	10,000	**21,250**
2	3	6	18,000	**21,750**
2	4	7	23,000	**22,500**
3	1	8	38,000	**22,125**
3	2	9	12,000	**22,625**
3	3	10	13,000	**24,125**
3	4	11	32,000	
4	1	12	41,000	

All methods are evaluated and analysed using error analysis. [Use excel]

$$\overline{D_t} = (D_{t-2} + D_{t+2} + \sum_{i=t-1}^{t+1} 2D_i)/8$$

$$t = 3,\ldots,10$$

$$\overline{D_3} = (D_1 + D_5 + 2[D_2 + D_3 + D_4])/8$$

$$\overline{D_3} = 1000.(8 + 10 + 2.[13 + 23 + 34])/8 = 19,750$$

Case Study (cont)

- Regress $\overline{D_t}$ vs t (t=3,...,10); i.e: $\overline{D_t} = L + tT$
- Using Excel → L=18,439 & T=524 → $\overline{D_t} = 18,439 + 524t$ (*)
- Recalculate $\overline{D_t}$ for all periods, t=1,...12 using (*).
- **Estimate Seasonal factors** $\overline{S_t} = \dfrac{D_t}{\overline{D_t}}$
- Calculate S for each quarter (mean):
  S1=(0.42+0.47+0.52)/3=0.47
  S2=0.68;S3=1.17;S4=1.67
- **Compute Forecasts**

$F_{13} = (L + 13T)S_{13} =$

(18,439+13.524)0.47=11,868

$F_{14} = 17,527$

$F_{15} = 30,770$

$F_{16} = 44,794$

Year	Quarter	Period t	Demand, Dt	Deseasonalized Demand	Computed $\overline{D_t}$	$\overline{S_t}$
1	2	1	8,000		18,963	0.42
1	3	2	13,000		19,487	0.67
1	4	3	23,000	19,750	20,011	1.15
2	1	4	34,000	20,625	20,535	1.66
2	2	5	10,000	21,250	21,059	0.47
2	3	6	18,000	21,750	21,583	0.83
2	4	7	23,000	22,500	22,107	1.04
3	1	8	38,000	22,125	22,631	1.68
3	2	9	12,000	22,625	23,155	0.52
3	3	10	13,000	24,125	23,679	0.55
3	4	11	32,000		24,203	1.32
4	1	12	41,000		24,727	1.66

Adaptive Forecasting Methods

- The estimates of L, T and S are updated after each demand observation
- The forecasts are calculated as follows:

$$F_{(t+l)} = (L_t + lT_t)S_{(t+l)} \qquad (6.2)$$

- Main Idea:
  1. Find initial values of L, T and S as in the static model.
  2. Use Equation (6.2) to find the forecast
  3. Estimate the error: E=F-D (see more later)
  4. Modify the estimate of L, T and S based on the error.
  5. Repeat steps 2-4 until demand in all future periods are obtained.

Moving Average

- No trend & no Seasonality (T=0; S=1)
- Average demand of most recent N periods
- Number of periods required (N) is critical:

(If N is too large, average over long period, N is too small the average just over last few).

Example: weekly demand of milk: 120, 127,114 and 122 litres over last 4 weeks. Assume demand in week 5 is 125 litres. Find forecast for week 5.

Using a 4-period moving average (N=4):

F5=(D1+...+D4)/4=120.75; Errror= F5-D5=4.25 litres; F6=(D2+...D5)/4 =122

Using a 3-period moving average (N=3):

F5=(D2+D3+D4)/3 = 122; Error=F5-D5=3 litres; F6=(D3+D4+D5)/3=120.3

Simple Exponential Smoothing

- No trend & no seasonality (T=0;S=1)→$(i.e., F_t = L_t)$
- Combine the importance of the last demand with the previous forecast
- Initially $F_1 = \sum_{i=1}^{i=n} D_i / n$
- Afterward we use:

$$F_t = \alpha D_{(t-1)} + (1-\alpha)F_{(t-1)} \qquad (6.3)$$

with $0 < \alpha < 1$ as the smoothing constant for the level.

Same example

Use $(\alpha = 0.1)$

$$F_1 = (D_1 + \ldots + D_4)/4 = 120.75$$

$$F_2 = \alpha D_1 + (1-\alpha)F_1 = 0.1(120) + 0.9(120.75) = 120.68$$

$$F_3 = \alpha D_2 + (1-\alpha)F_2 = 0.1(127) + 0.9(120.68) = 121.31$$

$$F_4 = 120.58; F_5 = \alpha D_4 + (1-\alpha)F_4 = 120.72$$

Trend-corrected exponential smoothing (Holt's Method)

- No seasonality but there is a trend
- Forecast based on previous level & trend: $F_{(t+l)} = L_{(t+l-1)} + T_{(t+l-1)}$
- Initially compute L,T using regression as in the static method and find $F_1 = L_0 + T_0$
- Update L and T as follows: $\begin{cases} L_{(t+l)} = \alpha D_{(t+l)} + (1-\alpha)[L_{(t+l-1)} + T_{(t+l-1)}] \\ T_{(t+l)} = \beta[L_{(t+l)} - L_{(t+l-1)}] + (1-\beta)T_{(t+l-1)} \end{cases}$
- Compute the new forecast $F_{(t+l+1)} = T_{(t+l)} + L_{(t+l)}$
- Repeat the updating till all forecasts are done
- Note: $0 < \alpha < 1$ is the smoothing constant for the level

  $0 < \beta < 1$ // // the trend

Example (Holt's method)

- Electronic manufacturer (MP3 player) recorded the last 6 months increasing demand of:

  8,4125; 8,752; 9,808; 10,413 and 11,961.

- Forecast month 7 using $\alpha = 0.1; \beta = 0.2$
- Using regression: $D_t = L_0 + tT_0$ we find $L_0 = 7,367; T_0 = 673$
- Initial forecast for month 1: $F_1 = L_0 + T_0 = 7,367 + 673 = 8,040$
- Revised forecast:

$$L_1 = \alpha D_1 + (1-\alpha)F_1 = 0.1(8,4125) + 0.9(8,040) = 8,078$$

$$T_1 = \beta(L_1 - L_0) + (1-\beta)T_0 = 0.2(8,078 - 7,367) + 0.9(673) = 681$$

$$F_2 = L_1 + T_1 = 8,759$$

$$L_2 = 8,755; T_2 = 680$$

$$L_3 = 9,393; T_3 = 672$$

$$L_4 = 10,039; T_4 = 666$$

$$L_5 = 10,676; T_5 = 661$$

$$L_6 = 11,399; T_6 = 673 \Rightarrow F_7 = T_6 + L_6 = 12,072$$

Trend & Seasonality Corrected Exponential Smoothing (Winter's Method)

- Both trend and seasonality exist
- Extension of Holt's method
- Forecasts: $F_{(t+l)} = (L_t + lT_t)S_{(t+l)}$
- Updating process of L,T and S:

$$\begin{cases} L_{(t+l)} = \alpha(\frac{D_{t+l}}{S_{t+l}}) + (1-\alpha)(L_{(t+l-1)} + T_{(t+l-1)}) \\ T_{t+l} = \beta(L_{(t+l)} - L_{(t+l-1)}) + (1-\beta)T_{(t+l-1)} \\ S_{(t+l+p)} = \gamma(\frac{L_{(t+l)}}{L_{(t+l)}}) + (1-\gamma)S_{(t+l)} \end{cases}$$

- Initially $F_{t+1} = (L_t + T_t)S_{(t+1)}$ with $L_0, T_0, (S_1, ..., S_p)$ found as in the static model.
- Repeat till the demand of all periods is computed.

Example (Tahoe salt)

- Consider demand data (Tahoe Salt) and forecast periods 1 & 2, using: $\alpha = 0.1; \beta = 0.2; \gamma = 0.1$
- Initial estimates: $L_0 = 18439; T_0 = 524; (S_1 = 0.47; S_2 = 0.68; S_3 = 1.17; S_4 = 1.67)$
- Forecast in period 1: $F_1 = (L_0 + T_0)S_1 = (18439 + 524)0.47 = 8013$
- Forecast error in period 1: $E_1 = F_1 - D_1 = 8913 - 8000 = 913$
- Revise the estimates of L1,T1 and S5:

$$L_1 = \alpha(D_1 / S_1) + (1-\alpha)(L_0 + T_0) = 0.1(8000 / 0.47) + (0.9)(18439 + 524) = 18769$$

$$T_1 = \beta(L_1 - L_0) + (1-\beta)T_0 = 0.2(18,769 - 18439) + 0.8(524) = 485$$

$$S_5 = \gamma(D_1 / L_1) + (1-\gamma)S_1 = 0.1(8000 / 18769) + 0.9(0.47) = 0.47$$

- Forecast in period 2: $F_2 = (L_1 + T_1)S_2 = (18769 + 485)0.68 = 13093$
- **For this problem, Winter's model (trend & seasonality) is most appropriate as the demand experiences trends & seasonality.**

Measures of forecasting error

- **Forecasting error in period t is:** $E_t = F_t - D_t$
- Error analysis helps in choosing the right forecasting method. Some measures are used.

(a) Mean Square Error (MSE): $MSE_n = \dfrac{\sum_{t=1}^{n} E_t^2}{n}$

[expect mean forecasted error →0

with var(error) →MSE and error →N(0,MSE)]

(b) Mean Absolute deviation (MAD): $MAD_n = \dfrac{\sum_{t=1}^{t=n} |E_t|}{n}$

[we expect the error to follow: N(0,1.25MAD)]

(c) Mean Absolute Percentage Error (MAPE): $MAPE_n = 100 \dfrac{\sum_{t=1}^{t=n} \left| \dfrac{E_t}{D_t} \right|}{n}$

(d) Bias: $Bias_n = \sum_{t=1}^{t=n} E_t$ **[If it is close to zero, truly random error (not biased, otherwise it is and action is needed.]**

(e) Tracking signal (TS): $TS_t = \dfrac{Bias_t}{MAD_t}$

[If -6<TS<6 , it is ok, else: <-6 (underestimate, >6 (overestimate), take action]

{similar to control chart of the errors, if outside range take action}

Basic Comparison

- The Naïve method- just consider the previous observation as your next estimate- equivalent to one period moving average. (i.e., F(t)=D(t-1))
- Use the Theil- U test

$$U = \frac{\sum_{t=1}^{n-1}\left[\frac{(F_{t+1}-D_{t+1})}{D_t}\right]^2}{\sum_{t=1}^{n-1}\left[\frac{(D_{t+1}-D_t)}{D_t}\right]^2}$$

Note:

- If U < 1 Forecast better than the Naïve
- If U=1 Forecast is as good as the Naïve
- If U> 1 Forecast is worse than the Naïve

Drawbacks: If the series fluctuates, that's worse. One way is to modify it to refer to using the previous one instead, F(t)=D(t-2) and check!

Case study (Tahoe Salt, p205→): adaptive

- **The demands (in tonnes of salt produced) of 3 years with 12 periods (4 quarters each)**
- **Aim to forecast the next 4 quarters.**

Year	Quarter	Period t	Demand, Dt
1	2	1	8,000
1	3	2	13,000
1	4	3	23,000
2	1	4	34,000
2	2	5	10,000
2	3	6	18,000
2	4	7	23,000
3	1	8	38,000
3	2	9	12,000
3	3	10	13,000
3	4	11	32,000
4	1	12	41,000

All methods are evaluated and analysed using error analysis.

[Use excel in lab]