Queen Mary
University of London

B. Sc. Examination by course unit 2013

MTH4101 Calculus II Sample Exam

Duration: 2 hours

Date and time: Summer 2013

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You should attempt all questions. Marks awarded are shown next to the questions.

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Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): R. Klages

Question 1

(a) Does the sequence

$$a_n = \frac{n}{5^n}$$

converge or diverge? Find the limit if the sequence is convergent. [7]

(b) Solve the differential equation

$$\frac{dy}{dx} = e^{x-y} \, .$$

[7]

(c) Consider the function

$$f(x,y) = \frac{y}{x} \, , \; x \neq 0 \, .$$

Show by using different paths of approach that the limit of f does not exist as $(x,y) \to (0,0)$. [7]

(d) Find the directional derivative of the function

$$f(x,y,z) = -7\cos(yz)e^x \, ,$$

at the point $(0,0,0)$, in the direction of the vector $\mathbf{A} = 4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$. [7]

(e) Find all the local maxima, local minima and saddle points of the function

$$f(x,y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4 \, .$$

[7]

(f) Find the sum of the series

$$\sum_{n=0}^{\infty} \left(\frac{(-1)^n}{2^n} - \frac{1}{4^n} \right) \, .$$

[7]

(g) Find the Taylor polynomials of order one and two for the function

$$f(x) = \ln(\cos x) \, ,$$

about the point $x = 0$. [7]

(h) Sketch the region of integration, and then reverse the order of integration, to evaluate the integral

$$\int_0^1 \int_y^1 x^2 e^{xy} \, dx \, dy \, .$$

[7]

Question 2 State the Integral Test for a series $\sum a_n$ with positive terms. Use this test to investigate the convergence of the series

$$\sum_{n=3}^{\infty} \frac{\ln(n+1)}{n+1} .$$

[11]

Question 3 Use the chain rule of partial differentiation to express $\partial w/\partial u$ and $\partial w/\partial v$ as functions of u and v for

$$w = xy + yz + xz, \quad x = u + v, \quad y = u - v, \quad z = uv.$$

Show that one obtains the same result by expressing w in terms of u and v directly. [11]

Question 4 Find the maximum value of $f(x, y) = 5 - x^2 - y^2$ on the line $x - 2y = 2$ by eliminating x in the expression for f. Show that one obtains the same result by using the Lagrange multiplier method. [11]

Question 5 Solve the system $u = 3x + 2y$, $v = x + 4y$ to find expressions for x and y in terms of u and v. Use these expressions to find the Jacobian $\partial(x, y)/\partial(u, v)$. Hence evaluate the integral

$$\int\int_R (3x + 2y)(x + 4y)\ \mathrm{d}x\ \mathrm{d}y$$

for the region R bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$ and $y = -(1/4)x$, $y = -(1/4)x + 1$. [11]

End of Paper