Queen Mary
University of London

B. Sc. Examination by course unit 2013

MTH4101: Calculus II

Duration: 2 hours

Date and time: 14 May 2013, 10:00h–12:00h

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You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): R. Klages, Y.Fyodorov

Question 1

(a) Consider the function

$$f(x,y) = \frac{y^2 - xy}{\sqrt{x} - \sqrt{y}}\ ,\ x, y > 0\ ,\ x \neq y\ .$$

Find the limit of f as $(x,y) \to (0,0)^+$. [7]

(b) Find all first-order and second-order partial derivatives of the function

$$f(x,y) = e^{3y}\cos x + \ln(2y) - e^x \sin y\ .$$

[7]

(c) Find the equations for the tangent plane and normal line at the point $P_0(1,-1,3)$ for the surface

$$x^2 + 2xy - y^2 + z^2 = 7\ .$$

[7]

(d) Obtain the limit as $n \to \infty$ for the sequence

$$a_n = \left(\frac{1}{n}\right)^{(1/\ln n)}\ .$$

[7]

(e) Use a suitable test to determine whether the series

$$\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$$

converges or diverges. [7]

(f) Find the Taylor series generated by

$$f(x) = x^3 - 2x + 1$$

at the point $a = 3$. [7]

(g) Sketch the region of integration, and then evaluate the triple integral

$$\int_1^e \int_0^2 \int_0^1 \frac{x^2 y}{z}\ dx\ dy\ dz\ .$$

[7]

(h) Solve the differential equation

$$\sqrt{x}\frac{dy}{dx} = e^{y+\sqrt{x}}\ ,\ x > 0\ ,$$

by giving the solution in implicit form. [7]

Question 2 Use the chain rule of partial differentiation to express dw/dt as a function of t for

$$w = \frac{x}{z^2} + \frac{y}{z^2}, \quad x = \cos^2 t, \quad y = \sin^2 t, \quad z = \frac{1}{t}.$$

Show that one obtains the same result by expressing w in terms of t directly. [11]

Question 3 Use the method of Lagrange multipliers to find the extreme points of the function

$$f(x, y, z) = x^2 + y^2 + z^2$$

subject to the condition

$$(x - 3)^2 + (y - 2)^2 + (z - 1)^2 = 1\,.$$

[11]

Question 4 State the Ratio Test for a series $\sum a_n$ with positive terms. Use this test to investigate the convergence of the series

$$\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}\,.$$

[11]

Question 5 Find all locations and values of the local maxima, local minima, and saddle points of the function

$$f(x, y) = 2x^2 + xy + 6x + 3y - 4\,.$$

[11]

End of Paper