Queen Mary
University of London

B. Sc. Examination by course unit 2012

MTH4101 Calculus II

Duration: 2 hours

Date and time: 8 May 2012, 10:00h–12:00h

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You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): R. Klages, R. Tavakol

Question 1

(a) If $z = x + iy$ is a complex number, find the expressions for $|z|$ and $|z - i|$ in terms of x and y, and hence describe the locus of the points in the Argand diagram for which $|z| = |z - i|$. [7]

(b) Use de Moivre's theorem to express $\cos 3\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$. [7]

(c) Consider the function

$$f(x, y) = \frac{y}{x}\,,\; x \neq 0\,.$$

Show by using different paths of approach that the limit of f does not exist as $(x, y) \to (0, 0)$. [7]

(d) Find the directional derivative of the function

$$f(x, y, z) = -7\cos(yz)e^{x}\,,$$

at the point $(0, 0, 0)$, in the direction of the vector $\mathbf{A} = 4\mathbf{i} + 3\mathbf{j} - 5\mathbf{k}$. [7]

(e) Find all the local maxima, local minima and saddle points of the function

$$f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y - 4\,.$$

[7]

(f) Sketch the region of integration, and then reverse the order of integration, to evaluate the integral

$$\int_0^1 \int_y^1 x^2 e^{xy}\, dx\, dy\,.$$

[7]

(g) Find the sum of the series

$$\sum_{n=0}^{\infty} \left(\frac{(-1)^n}{2^n} - \frac{1}{4^n} \right)\,.$$

[7]

(h) Find the Taylor polynomials of order one and two for the function

$$f(x) = \ln(\cos x)\,,$$

about the point $x = 0$. [7]

Question 2

(a) Find all solutions to the equation $z^3 + 27 = 0$ in polar form, and sketch their locations in an Argand diagram. [6]

(b) Use de Moivre's theorem to simplify, as much as possible, the fraction

$$\frac{(\cos 5\theta + i \sin 5\theta)^2}{(\cos 2\theta - i \sin 2\theta)^3}.$$

[5]

Question 3 Use the chain rule of partial differentiation to express $\partial w/\partial u$ and $\partial w/\partial v$ as functions of u and v for

$$w = xy + yz + xz, \quad x = u + v, \quad y = u - v, \quad z = uv.$$

Show that one obtains the same result by expressing w in terms of u and v directly. [11]

Question 4 Find the maximum value of $f(x, y) = 5 - x^2 - y^2$ on the line $x - 2y = 2$ by eliminating x in the expression for f. Show that one obtains the same result by using the Lagrange multiplier method. [11]

Question 5 Solve the system $u = 3x + 2y$, $v = x + 4y$ to find expressions for x and y in terms of u and v. Use these expressions to find the Jacobian $\partial(x, y)/\partial(u, v)$. Hence evaluate the integral

$$\int\int_R (3x + 2y)(x + 4y)\, \mathrm{d}x\, \mathrm{d}y$$

for the region R bounded by the lines $y = -(3/2)x + 1$, $y = -(3/2)x + 3$ and $y = -(1/4)x$, $y = -(1/4)x + 1$. [11]

End of Paper