Queen Mary
University of London

B. Sc. Examination by course unit 2011

MTH4101 Calculus II

Duration: 2 hours

Date and time: 5 May 2011, 1430h–1630h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): C. D. Murray, C. B. Agnor

Question 1

(a) If $z = x + iy$ is a complex number, find expressions for $|z|$ and $|z - 2|$ in terms of x and y, and hence shade the region in the Argand diagram for which $|z| \leq |z - 2|$. [7]

(b) Express the complex number $z = \sqrt{3} + i$ in polar form. Hence use de Moivre's theorem to find the cube roots of z and sketch their locations on an Argand diagram. [7]

(c) Find all the first-order and second-order partial derivatives of the function $f(x, y) = \sin(x^2 y) - e^x \cos y$. [7]

(d) Find the equation of the tangent plane and the parametric equations of the normal line at the point $(2, 1, 4)$ on the surface defined by $z - x^2 y = 0$. [7]

(e) Find the volume of the solid that is bounded from above by the plane $z = 4 - x - y$ and from below by the rectangle $0 \leq x \leq 1$, $0 \leq y \leq 2$. [7]

(f) Find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$ for the transformation $x = 4u + v$, $y = u - 2w$, $z = v + w$. [7]

(g) Use the Sandwich Theorem for Sequences to find

$$\lim_{n \to \infty} \frac{\sin n}{n} .$$

[7]

(h) Find the sum of the series

$$\sum_{k=1}^{\infty} (-1)^{k-1} \frac{3}{5^{k-1}} .$$

[7]

Question 2

(a) Show that

$$(1+i)^i = e^{-\left(\frac{\pi}{4}+2k\pi\right)}\left\{\cos(\ln\sqrt{2}) + i\sin(\ln\sqrt{2})\right\} .$$

[5]

(b) By writing $e^{ax}\cos bx$ as the real part of $e^{(a+ib)x}$, where a and b are real numbers, use complex numbers to show that

$$\int e^{ax}\cos bx\,\mathrm{d}x = \frac{e^{ax}}{a^2+b^2}\left(a\cos bx + b\sin bx\right) .$$

[6]

Question 3 Use the method of Lagrange multipliers to find the maximum and minimum values of the function

$$f(x,y,z) = 2x + y - 2z$$

subject to the constraint $x^2+y^2+z^2 = 4$. [11]

Question 4

(a) Sketch the region of integration and then reverse the order of integration to evaluate the integral

$$\int_0^4\int_{\sqrt{y}}^2 e^{x^3}\,\mathrm{d}x\,\mathrm{d}y .$$

[6]

(b) Sketch the region of integration and then transform to polar coordinates to evaluate the integral

$$\int_0^2\int_0^{\sqrt{4-x^2}} (x^2+y^2)\,\mathrm{d}y\,\mathrm{d}x .$$

[5]

Question 5

(a) Derive the Maclaurin series for the functions (i) e^x and (ii) $\cos x$ from first principles. Your answers should include terms up to x^4. [6]

(b) Use your results from part (a) to write down power series for e^{-x^2} and $\cos(x^2)$ including terms up to x^4. Hence derive a power series for

$$f(x) = e^{-x^2}\cos(x^2)$$

including terms up to x^4. [5]

End of Paper