Queen Mary
University of London

B. Sc. Examination by course unit 2010

MTH4101 Calculus II

Duration: 2 hours

Date and time: 14 May 2010, 1430h–1630h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

Candidates should note that the Examination and Assessment Regulations state that possession of unauthorized materials by any candidate who is under examination conditions is an assessment offence. Please check your pockets now for any notes that you may have forgotten that are in your possession. If you have any, then please raise your hand and give them to an invigilator now.

Exam papers must not be removed from the examination room.

Examiner(s): C. D. Murray, R. P. Nelson

Question 1

(a) Use de Moivre's Theorem to express $\sin 4\theta$ in terms of powers of $\cos\theta$ and $\sin\theta$. [7]

(b) Find the cube roots of the complex number $8+8\,i$ and plot them on an Argand diagram. [7]

(c) Locate and determine the nature of all the critical points of the function

$$f(x, y) = 3x^2 - 2xy + y^2 - 8y\,.$$

[7]

(d) Find the directional derivative of the function $f(x, y) = e^{xy}$ at the point $(-2, 0)$ in the direction of the unit vector that makes an angle of $\pi/3$ with the positive x-axis. [7]

(e) Evaluate the double integral

$$\iint_R y^2 x \,\mathrm{d}A$$

where R is the region defined by $-3 \le x \le 2$, $0 \le y \le 1$. [7]

(f) Sketch the region of integration and use polar coordinates to evaluate the double integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,\mathrm{d}y\,\mathrm{d}x\,.$$

[7]

(g) Find the sum of the series

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{7}{9^{n-1}}\,.$$

[7]

(h) Use the ratio test to determine whether the series

$$\sum_{k=3}^{\infty}\frac{(2k)!}{4^k}$$

converges or diverges. [7]

Question 2 Use the method of Lagrange multipliers to find the minimum value of the function

$$f(x, y, z) = xy + 2xz + 2yz$$

subject to the constraint $xyz = 32$. [11]

Question 3 Sketch the region R (in $x \geq 0$, $y \geq 0$) enclosed by the curves $y = x$, $y = 2x$, $xy = c^2$, $xy = 2c^2$ (where $c \neq 0$ is a constant). By changing to new variables $u = u(x, y) = y/x$ and $v = v(x, y) = xy$, calculate the Jacobian $\partial(u, v)/\partial(x, y)$ and hence evaluate the integral

$$\iint_R (x^2 + y^2)\, \mathrm{d}x\, \mathrm{d}y\,.$$

[11]

Question 4 Find the first four Taylor polynomials for the function $f(x) = \ln x$ about the value $x = 2$. [11]

Question 5

(a) Use the integral test to find the values of p for which the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

converges or diverges. [6]

(b) Use the integral test with the substitution $x = \tan y$ to determine whether the series

$$\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{1 + n^2}$$

converges or diverges. [5]

End of Paper