Queen Mary
University of London

B. Sc. Examination by course unit 2013

MTH4100 Calculus 1

Duration: 2 hours

Date and time: 7 May 2013, 14:30h–16:30h

Apart from this page, you are not permitted to read the contents of this question paper until instructed to do so by an invigilator.

You should attempt all questions. Marks awarded are shown next to the questions.

Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.

Complete all rough workings in the answer book and cross through any work which is not to be assessed.

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Examiner(s): Prof. B. Jackson

Question 1 (a) Find the natural domain of

$$f(x) = \frac{x^2 - 4}{x + 2}$$

and determine whether $f(x)$ is continuous at $x = -2$. [5 marks]

(b) Find the limit

$$\lim_{x \to 0} \frac{1 - \cos(2x)}{(2x)^2} \, .$$

[5 marks]

(c) Find the derivative, $g'(x)$, of

$$g(x) = \ln\left(\sin(x^2)\right) \, .$$

[6 marks]

(d) Find the horizontal, vertical and oblique asymptotes, if any, of

$$f(x) = \frac{x^2 + x + 1}{x^2 + x - 3} \, .$$

[6 marks]

(e) Given that

$$\cos y = \sin 2x \, ,$$

and $y = y(x)$, find the values of dy/dx and d^2y/dx^2 when $x = \pi/6$. [8 marks]

(f) Find the area enclosed by the curves $x = 0$, $y = 3$ and $y = \sqrt{x}$. [7 marks]

(g) Evaluate

$$\int x^2 \ln x \, dx \, .$$

[8 marks]

Question 2 Consider the curve $y = f(x)$ for the function

$$f(x) = x^3 - \frac{1}{x}\,.$$

(a) Identify the domain of f and any symmetries the function may have. [3 marks]

(b) Find $f'(x)$ and $f''(x)$. [2 marks]

(c) Find the critical points for f, and identify the function's behaviour at each critical point. [5 marks]

(d) Find where the curve is increasing and where it is decreasing. [3 marks]

(e) Find the inflection points for f, if any occur, and determine the concavity of the curve. [5 marks]

(f) Determine the behavior of $f(x)$ as $x \to \pm\infty$ and identify any asymptotes. [2 marks]

(g) Plot key points, such as intercepts, critical points, and points of inflection, and sketch the curve. [5 marks]

Question 3 (a) State the definition of the derivative of the function $f(x)$ with respect to the variable x. [3 marks]

(b) Differentiate from first principles, $f(x) = \dfrac{1}{x+2}$, that is, by using the above definition of the derivative. [6 marks]

(c) Find the values of x for which f' is defined. Determine the values of x for which the function f is continuous (give reasons for your answer). [6 marks]

Question 4 (a) Explain what condition a function f must satisfy in order to have an inverse and define the inverse function of f when this condition is satisfied. [4 marks]

(b) Define the natural logarithm function $\ln x$ and give its domain and range. Explain why its derivative is $1/x$. Deduce that its inverse function, $\exp x$, satisfies

$$\frac{d}{dx}\exp x = \exp x\,.$$

[7 marks]

(c) Evaluate

$$\int \cos x\, \exp(\sin x)\, dx\,.$$

[4 marks]

End of Paper