Queen Mary
University of London

B.Sc. EXAMINATION BY COURSE UNITS

MAS125 Calculus II

16 May 2008, 10.00 – 12.00

The duration of this examination is 2 hours.

You should attempt all questions. Marks awarded are shown next to the questions. Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

Candidates must not remove the question paper from the examination room.

YOU ARE NOT PERMITTED TO START READING THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

1. Marks are awarded for partial answers, so you should show your workings.

(a) [7 marks] If $z = x + iy$ is a complex number, find and sketch the region in the Argand diagram for which $|z + 1| \geq |z|$ and interpret this geometrically.

(b) [7 marks] Find the cube roots of the complex number $-5\sqrt{2}+5\sqrt{2}i$ and plot these on an Argand diagram.

(c) [7 marks] Find the sum of the series:

$$\sum_{n=0}^{\infty} \left(\frac{2}{5^n} - \frac{(-1)^n}{3^n} \right).$$

(d) [7 marks] Find the first four terms (i.e. up to and including terms of order x^6) of the binomial series of $(1 - x^2)^{1/2}$.

(e) [7 marks] Find the radius and interval of convergence for the series

$$\sum_{n=1}^{\infty} \frac{x^n}{n\sqrt{n}\, 3^n}$$

being careful to specify the behaviour at the end points of the interval.

(f) [7 marks] Find all first-order and second-order derivatives of the function $f(x, y) = ye^x - x \sin y + x^2 - y^2$.

(g) [7 marks] Find the equation of the tangent plane and the equation of the normal line at the point $P_0(1, 0, 1)$ on the surface (x, y, z) such that $3z + x^2 = 4$.

(h) [7 marks] Evaluate the integral

$$\int\int_R e^{x-y}\, \mathrm{d}x\, \mathrm{d}y$$

where R is the triangular region bounded by the lines $x = 0$, $y = 0$ and $y + x = 1$.

[Next question overleaf]

2. [11 marks] Use the integral test to find the values of p for which the series.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

converges. Explain why the ratio test cannot be used to determine the convergence of this series.

3. [11 marks] Use the method of Lagrange multipliers to find the extreme points of the function $f(x,y,z) = x^2+y^2+z^2$ subject to the condition $(x-1)^2+(y-2)^2+(z-3)^2 = 4$.

4. [11 marks] Find the linearisation $L(x,y,z)$ of the function $f(x,y,z) = xz + 2yz - 3$ at the point $P_0(1,1,2)$ and hence find an upper bound for the error E in approximating $f(x,y,z)$ by $L(x,y,z)$ over the rectangle $|x-1| \leq 0.1$, $|y-1| \leq 0.1$, $|z-2| \leq 0.2$.

5. [11 marks] Sketch the region of integration of the double integral

$$\int_0^1 \int_{\sqrt{y}}^1 y \, \sin x^5 \, \mathrm{d}x \, \mathrm{d}y \, .$$

By reversing the order of the integration, evaluate the integral.

[End of examination paper]