Calculus III
Test

11 Nov 2010

Time allowed: 40 min

Name: **Student number:**

There are **ten** questions. All questions are multiple-choice. Please enter answers with an X in the appropriate box. If you want to change an answer, solid-fill in the old answer, and use an X for your final answer.

Blank sheets are provided for rough work.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks. The maximum mark is 35.

There are several equivalent but re-ordered versions of this paper. Do not be concerned if other people appear to have a different paper.

1. Two points are given by position vectors $\mathbf{a}$ and $\mathbf{b}$. The vector equation for the straight line through these two points is:

☐ $(\mathbf{r} - \mathbf{a}) \cdot \mathbf{b} = 0$

☐ $\mathbf{r} = \mathbf{a} + t\,(\mathbf{b} - \mathbf{a})$

☐ $\mathbf{r} = t\,(\mathbf{a} + \mathbf{b})$

2. The parametric equation

$$\mathbf{r}(t) = a\cos t\,\mathbf{i} + a\sin t\,\mathbf{j} + b\,t\,\mathbf{k}$$

represents an

☐ Ellipse

☐ Circle

☐ Helix

3. The arc-length of the curve $y = x^3$ from (0,0) to (1,1) is given by:

☐ $\int_0^1 \sqrt{1 + 9x^4}\,dx$

☐ $\int_0^1 \sqrt{1 + 3x^2}\,dx$

☐ $\int_0^1 \sqrt{x + x^3}\,dx$

4. A parallelepiped has three of its sides given by $\mathbf{a} = (3, 2, 1)$ and $\mathbf{b} = (2, -2, 1)$ and $\mathbf{c} = (2, 1, 0)$. The volume of the parallelepiped is:

☐ 5

☐ 7

☐ 9

5. The vector field $\mathbf{F} = (3x^2 + xy)\,\mathbf{i} + (2y + z)\,\mathbf{j} + (xy^2 + z)\,\mathbf{k}$. The divergence $\nabla \cdot \mathbf{F}$ is:

☐ $(6x + y)\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

☐ $6x + y + y^2 + 1$

☐ $6x + y + 3$

Continued overleaf

6. The equation of the tangent plane to the surface $xy + 2x^2z = 24$ at the point (3,2,1) is:

- ☐ $3x + 2y + z = 14$
- ☐ $y + 12z = 14$
- ☐ $x + y + 2x^2 + 4xz = 35$
- ☐ $14x + 3y + 18z = 66$

7. A surface is defined by two parameters u, v by $\mathbf{r} = \mathbf{r}(u, v)$. Which of the following is true ?

- ☐ $\frac{\partial \mathbf{r}}{\partial u}$ is normal to the surface.
- ☐ $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$ is tangent to the surface.
- ☐ $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$ is normal to the surface.
- ☐ $\frac{\partial \mathbf{r}}{\partial u}$ and $\frac{\partial \mathbf{r}}{\partial v}$ are perpendicular to each other.

8. The vector field
$\mathbf{F} = (x^3,\ y^2 + 2z,\ y + z)$.
The curl $\nabla \times \mathbf{F}$ is:

- ☐ $-\mathbf{i}$
- ☐ $3\,\mathbf{i}$
- ☐ $3x^2\,\mathbf{i} + 2y\,\mathbf{j} + \mathbf{k}$
- ☐ $3\,\mathbf{i} + 2\,\mathbf{j} + \mathbf{k}$

9. For a given scalar field U, the vector field $\mathbf{G}$ is defined by $\mathbf{G} = \nabla U$. Which of the following is definitely true ?

- ☐ $\nabla \times \mathbf{G} = \mathbf{0}$
- ☐ $\nabla \cdot \mathbf{G} = 0$
- ☐ $\nabla \times \mathbf{G} = \nabla^2 U$
- ☐ $\mathbf{G} \cdot \mathbf{G} = \nabla(U^2)$

10. The vector field $\mathbf{F} = z\,\mathbf{i} + 2y\,\mathbf{k}$. The line integral $\int \mathbf{F}.d\mathbf{r}$ along the curve $\mathbf{r} = t\,\mathbf{i} + 2t\,\mathbf{j} + t^2\,\mathbf{k}$ from (0,0,0) to (1,2,1) is:

- ☐ $\frac{5}{3}$
- ☐ 3
- ☐ $\frac{7}{3}$
- ☐ 5

End of test