Calculus III Test

11 Nov 2010

Time allowed: 40 min

Student number:

Name:

There are **ten** questions. All questions are multiple-choice. Please enter answers with an X in the appropriate box. If you want to change an answer, solid-fill in the old answer, and use an X for your final answer.

Blank sheets are provided for rough work.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks. The maximum mark is 35.

There are several equivalent but re-ordered versions of this paper. Do not be concerned if other people appear to have a different paper.

1. Two points are given by position vectors **a** and **b**. The vector equation for the straight line through these two points is:

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{b} = 0$$

$$\mathbf{r} = \mathbf{a} + t (\mathbf{b} - \mathbf{a})$$

$$\mathbf{r} = t (\mathbf{a} + \mathbf{b})$$

2. The parametric equation

$$\mathbf{r}(t) = a\cos t \,\mathbf{i} + a\sin t \,\mathbf{j} + b\,t\,\mathbf{k}$$

represents an

- Ellipse
- Circle
- Helix
- **3.** The arc-length of the curve $y = x^3$ from (0,0) to (1,1) is given by:

 $\Box \int_0^1 \sqrt{1+9x^4} \, dx$ $\Box \int_0^1 \sqrt{1+3x^2} \, dx$ $\Box \int_0^1 \sqrt{x+x^3} \, dx$

4. A parallelepiped has three of its sides given by $\mathbf{a} = (3, 2, 1)$ and $\mathbf{b} = (2, -2, 1)$ and $\mathbf{c} = (2, 1, 0)$. The volume of the parallelepiped is:

5
7
9

5. The vector field $\mathbf{F} = (3x^2 + xy)\mathbf{i} + (2y+z)\mathbf{j} + (xy^2+z)\mathbf{k}$. The divergence $\nabla \cdot \mathbf{F}$ is:

$$(6x+y)\mathbf{i} + 2\mathbf{j} + \mathbf{k}$$

$$6x + y + y^2 + 1$$

$$6x + y + 3$$

Continued overleaf

6. The equation of the tangent plane to the surface $xy + 2x^2z = 24$ at the point (3,2,1) is:

$$3x + 2y + z = 14$$

$$y + 12z = 14$$

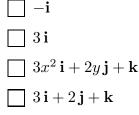
$$x + y + 2x^{2} + 4xz = 35$$

$$14x + 3y + 18z = 66$$

7. A surface is defined by two parameters u, v by $\mathbf{r} = \mathbf{r}(u, v)$. Which of the following is true ?

 $\Box \quad \frac{\partial \mathbf{r}}{\partial u} \text{ is normal to the surface.}$ $\Box \quad \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \text{ is tangent to the surface.}$ $\Box \quad \frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v} \text{ is normal to the surface.}$ $\Box \quad \frac{\partial \mathbf{r}}{\partial u} \text{ and } \frac{\partial \mathbf{r}}{\partial v} \text{ are perpendicular to each other.}$

8. The vector field $\mathbf{F} = (x^3, y^2 + 2z, y + z).$ The curl $\nabla \times \mathbf{F}$ is:



9. For a given scalar field U, the vector field **G** is defined by $\mathbf{G} = \nabla U$. Which of the following is definitely true ?

$$\nabla \times \mathbf{G} = \mathbf{0}$$
$$\nabla \cdot \mathbf{G} = 0$$
$$\nabla \times \mathbf{G} = \nabla^2 U$$
$$\mathbf{G} \cdot \mathbf{G} = \nabla(U^2)$$

 $\frac{5}{3}$

3

 $\frac{7}{3}$

5

10. The vector field $\mathbf{F} = z \mathbf{i} + 2y \mathbf{k}$. The line integral $\int \mathbf{F} d\mathbf{r}$ along the curve $\mathbf{r} = t \mathbf{i} + 2t \mathbf{j} + t^2 \mathbf{k}$ from (0,0,0) to (1,2,1) is:

End of test