Calculus III
Test

11 Nov 2009

Time allowed: 40 min

Name: **Student number:**

All questions are multiple-choice. Please enter answers by marking the appropriate box. Blank sheets are provided for rough work.

The maximum mark is 35.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks.

There are several equivalent but re-ordered versions of this paper. Do not be concerned if other people appear to have a different paper.

1. Let **a** and **b** be two constant vectors. The vector equation $(\mathbf{r}-\mathbf{a}).\mathbf{b}=0$ represents

☐ The line through the point **b** parallel to **a**

☐ The plane through the point **b** perpendicular to **a**

☐ The plane through the point **a** perpendicular to **b**

2. The vector field $\mathbf{F} = x^2\,\mathbf{i} + xyz\,\mathbf{j} + z^2\,\mathbf{k}$. The divergence $\nabla.\mathbf{F}$ is

☐ $2x + xz + 2z$

☐ $2x\mathbf{i} + xz\mathbf{j} + 2z\mathbf{k}$

☐ $2x + xz + yz + xy + 2z$

3. The equation

$$\frac{x^2}{4} + \frac{y^2}{9} = 1 + \frac{z^2}{16}$$

represents an

☐ Ellipsoid

☐ Hyperboloid

☐ Paraboloid

4. The arc-length of the parabola $y = x^2$ from (0,0) to (1,1) is given by

☐ $\int_0^1 \sqrt{1+4x^2}\,dx$

☐ $\int_0^1 \sqrt{1+2x}\,dx$

☐ $\int_0^1 x + x^2\,dx$

5. A parallelepiped has three of its sides given by $\mathbf{a} = (0,-1,2)$ and $\mathbf{b} = (2,1,0)$ and $\mathbf{c} = (2,2,1)$. The volume of the parallelepiped is

☐ 2

☐ 15

☐ 6

Continued overleaf

6. The equation of the tangent plane to the surface $x^2 + y^2 + z^2 + xyz = 20$ at the point (1,2,3) is:

- ☐ $x + 2y + 3z = 14$
- ☐ $8x + 7y + 8z = 46$
- ☐ $x + y + z = 6$
- ☐ $x^2 + y^2 + z^2 = 14$

7. Let U be any scalar field, and $\mathbf{F}$ be a solenoidal vector field. The divergence $\nabla.(U\mathbf{F})$ is :

- ☐ $U(\nabla.\mathbf{F})$
- ☐ 0
- ☐ $(\nabla U).\mathbf{F}$
- ☐ $U(\nabla \times \mathbf{F})$

8. The vector field $\mathbf{F} = z\,\mathbf{i} + y\,\mathbf{j} + x\,\mathbf{k}$. The unit cube is defined by the volume $0 \leq x, y, z \leq 1$. The surface integral $\int \mathbf{F}.\mathbf{n}\,dS$ over all six faces of the unit cube, with $\mathbf{n}$ the outward unit normal, is

- ☐ 1
- ☐ $\sqrt{3}$
- ☐ 0
- ☐ 6

9. The vector field $\mathbf{F} = (x^2 - y,\, y^2 + x,\, z^2)$. The curl $\nabla \times \mathbf{F}$ is

- ☐ $2x\mathbf{i} + 2y\mathbf{j} + 2z\mathbf{k}$
- ☐ $2\mathbf{k}$
- ☐ $2(x + y)\mathbf{i} - 2(y + z)\mathbf{j} + 2(z + x)\mathbf{k}$
- ☐ $\mathbf{0}$

10. The vector field $\mathbf{F} = z\,\mathbf{i}{+}2x\,\mathbf{k}$. The line integral $\int \mathbf{F}.d\mathbf{r}$ along the line $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ from (0,0,0) to (1,1,1) is:

- ☐ $\sqrt{3}$
- ☐ $\frac{3}{5}$
- ☐ 3
- ☐ $\frac{7}{4}$

End of test