Calculus III
Test

Nov 7, 2008

Time allowed: 40 min

Name: **Student number:**

All questions are multiple-choice. Please enter answers by marking the appropriate box. Blank sheets are provided for rough work.

The maximum mark is 35.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks.

There are several equivalent but not identical versions of this paper. Do not be concerned if others appear to have a different paper.

1. The line through a point at position $\mathbf{a}$ and parallel to a vector $\mathbf{n}$ is represented by the equation

☐ $(\mathbf{r}-\mathbf{a}).\mathbf{n}=0.$

☐ $(\mathbf{r}+\mathbf{a})\times\mathbf{n}=\mathbf{0}.$

☐ $\mathbf{r}=\mathbf{a}+t\mathbf{n}.$

2. The surface described by

$$x^2+z^2=9+y$$

is a

☐ sphere.

☐ paraboloid.

☐ hyperboloid.

3. $\mathbf{F}=(6x^2+2xy)\mathbf{i}+(2y+x^2z)\mathbf{j}+(xy^3+z)\mathbf{k}$ is a vector field. Its divergence $\nabla.\mathbf{F}$ is

☐ $(12x+2y)\mathbf{i}+2\mathbf{j}+\mathbf{k}.$

☐ $12x+2y+3.$

☐ $14x+2y+3+2xz+x^2+3xy^2+y^3.$

4. The Divergence Theorem says that under certain conditions

$$\int_{\mathcal{D}}\nabla.\mathbf{F}\,\mathrm{d}V=\int_{\mathcal{S}}\mathbf{F}.\mathbf{n}\,\mathrm{d}S$$

for a vector field $\mathbf{F}$ defined in a volume $\mathcal{D}$ enclosed by a surface $\mathcal{S}$. Which of the following conditions can be omitted when stating the Theorem?

☐ $\mathcal{S}$ is a closed oriented surface.

☐ The normal $\mathbf{n}$ on $\mathcal{S}$ is taken outward from $\mathcal{D}$.

☐ $\mathcal{S}$ is simple (i.e. is a single piece and does not cross itself).

5. One of the following is not necessarily true for a conservative vector field $\mathbf{F}$. Which?

☐ $\nabla\cdot\mathbf{F}=0.$

☐ $\nabla\times\mathbf{F}=\mathbf{0}.$

☐ $\mathbf{F}=\nabla\phi$ for some scalar field ϕ.

Continued overleaf

6. The expression

$$r = \sqrt{\cos(2\phi)}(\,\mathbf{i}\cos\phi + \mathbf{j}\sin\phi)$$

with $0 \leq \phi \leq \infty$ represents a curve in the x, y plane which is

- [ ] a circle.
- [ ] a heart shape, without a cusp (cardioid).
- [ ] an ellipse.
- [ ] a figure of eight (lemniscate).

7. $V = xy + 2x^2z$. The value of ∇V at the point $(-2, 3, 1)$ is

- [ ] $-5\mathbf{i} - 2\mathbf{j} + 8\mathbf{k}$.
- [ ] $-5\mathbf{i} - 2\mathbf{j} - 8\mathbf{k}$.
- [ ] 1.
- [ ] -15.

8. The curl of the vector field $\mathbf{F} = (x^2 - 3z,\, y,\, xz)$ is

- [ ] $2x\mathbf{i} + \mathbf{j} + x\mathbf{k}$.
- [ ] $(z + 3)\mathbf{j}$.
- [ ] $3x + 1$.
- [ ] $-(z + 3)\mathbf{j}$.

9. If $\mathbf{F} = z\mathbf{i} - y\mathbf{j} + x\mathbf{k}$, the integral $\int \mathbf{F}.\mathbf{dr}$ along the curve $\mathbf{r} = t\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ from (0, 0, 0) to (1, 1, 1) is

- [ ] 1/2.
- [ ] 1/5.
- [ ] 2/3.
- [ ] 6.

10. The surface of a sphere is parametrized as

$$\mathbf{r} = a(\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ \sin\phi).$$

The surface area element in terms of the coordinates $(\theta,\ \phi)$ is

- [ ] $a\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathbf{r}$.
- [ ] $a\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathbf{r}$.
- [ ] $-a\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathbf{r}$.
- [ ] $-a\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi\,\mathbf{r}$.

End of test