Calculus III
Answers to and comments on the 2008 test
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Referring to the test version as on the course web page, the answers were

1	3
2	2
3	2
4	3
5	1
6	4
7	1
8	4
9	1
10	3

Comments and details

111 scripts were received and the average score was approximately 63%. This was significantly better than last year and I was rather pleased with the result, though upset that somehow a misprint in question 10 slipped through until half-way through the test. Unfortunately I did not have any way to check who (apart from two students who specifically mentioned it to me) lost time by this. Most students did not answer, maybe for that reason. The only fair way I can see of making any amends is to scale as if the totals were out of 33.3... rather than 35, which improves most scores a bit.

4 questions were got right by over 75 students, namely the questions on surfaces, grad, div and curl, and another 3, on lines and planes, conservative fields, and line integrals by well over half the students, which is encouraging. 50 students scored over 70%.

Here are details for individual questions. In the performance analysis answer 0 means no answer, 1 means the first choice, and so on.

1. Answer 1 is the equation for a plane. Answer 2 is an incorrect equation for a line (wrong sign before **a**)

Performance: 0:7, 1:25, 2:13, 3:66. Generally well answered.

2. The similar equations for a sphere or hyperboloid respectively would be $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 = z^2 + 4$.

Performance: 0:7, 1:9, 2:77, 3:18. Enormous improvement over past years

3. The first answer catches those who forget that the divergence must be a vector. The last is in case somebody differentiates each component with respect to all three variables.

Performance: 0:1, 1:15, 2:92, 3:3. Very well answered, the best of all the questions.

4. This and the next are much simplified (compared with last year) ways to test knowledge of the main theorems. I discovered that some students did not know the word 'omit'. The answer could have been mistaken as a requirement because it is similar to a requirement on curves in Stokes's theorem.

Performance: 0:27, 1:23, 2:17, 3:44. I expected this would be more searching than some other questions.

5. The test revision advice specifically mentions conditions 2 and 3. Conservative fields may or may not obey 1.

Performance: 0:15, 1:66, 2:17, 3:13. As this type of question is less familiar, I was pleased by the answers.

6. If one thinks of this as a curve in polar coordinates, it has $r = \sqrt{\cos(2\phi)}$ and angular coordinate ϕ. A circle would have r =constant, the cardioid would have something like $r = 1{+}\cos\phi$, see Thomas figure 10.50, and an ellipse would have $r = \sqrt{a^2\cos^2\phi + b^2\sin^2\phi}$ (which is not a multiple of $\cos(2\phi)$ since neither a^2 and b^2 are both positive). The lemniscate is shown in Figure 10.55 in Thomas.

Performance: 0:36, 1:8, 2:24, 3:14, 4:29. Although it is covered in the first year, understanding curves seems to be a real weakness and the correct answer was only marginally the most popular of the 4 while many left this one out. I was surprised how popular the third answer was.

7. Answer 2 arises from losing a minus. Answers 3 and 4 relate to 1 and 2 by adding the components together, forgetting that a gradient must be a vector.

Performance: 0:4, 1:85, 2:6, 3:11, 4:5. Very well answered.

8. Answer 1 comes from taking

$$\frac{\partial F_x}{\partial x}\mathbf{i} + \frac{\partial F_y}{\partial y}\mathbf{j} + \frac{\partial F_z}{\partial z}\mathbf{k}.$$

Answer 2 comes from losing an overall sign in the $\mathbf{j}$ component. Answer 3 is the divergence, rather than the curl.

Performance: 0:3, 1:12, 2:19, 3:0, 4:77. Well answered. Obviously most of those who got it wrong lost a sign in the evaluation.

9. On the curve $\mathrm{d}\mathbf{r} = \mathbf{r}_t\,\mathrm{d}t = (\mathbf{i} + 2t\,\mathbf{j} + 3t^2\,\mathbf{k})\mathrm{d}t$ and $\mathbf{F} = t^3\mathbf{i} - t^2\mathbf{j} + t\mathbf{k}$, so the dot product is $2t^3\,\mathrm{d}t$. The limits on t are 0 and 1. The integral becomes

$$\int_0^1 (2t^3)\mathrm{d}t = [\tfrac{1}{2}t^4]_0^1 = \tfrac{1}{2}\ .$$

The wrong answers arise as follows. Answer 2 comes from using $\mathbf{F}.\mathbf{r}\,\mathrm{d}t$ rather than $\mathbf{F}.\mathrm{d}\mathbf{r}$. Answer 3 comes from using $\mathrm{d}\mathbf{r}$ rather than $\mathbf{r}$ when substituting in $\mathbf{F}$. Answer 4 comes from differentiating $2t^3$ rather than integrating.

Performance: 0:25, 1:71, 2:3, 3:6, 4:7. Much improved on last year's line integral.

10. It is a pity that the first try at setting a calculation of $\mathrm{d}\mathbf{S}$ was marred by the misprint. The formula should have read

$$\mathbf{r} = a(\cos\theta\cos\phi,\ \cos\theta\sin\phi,\ \sin\theta).$$

Candidates were told this about half-way through the test. It would be interesting to know if more would have given an answer if this had not happened. For the "distractors" (wrong answers), 1 was correct up to sign, and 2 and 4 were based on the more usual parametrization (where sine and cosine of θ are swapped) for which 2 would be correct .

Performance: 0:63, 1:11, 2:11, 3:21, 4:5. Hard to interpret the outcome due to the misprint, though the correct answer was the most popular for those who answered at all.