Calculus III
Test

Nov 9, 2007

Time allowed: 40 min

Name: **Student number:**

All questions are multiple-choice. Please enter answers by marking the appropriate box. Blank sheets are provided for rough work.

The maximum mark is 35.

Questions 1–5 carry 3 marks each, and 6–10 carry 4 marks each. Each question left unanswered gets 1 mark. Incorrect answers get no marks.

There are several equivalent but not identical versions of this paper. Do not be concerned if others appear to have a different paper.

1. The plane through a point at position $\mathbf{a}$ and perpendicular to a vector $\mathbf{n}$ is represented by the equation

- ☐ $\mathbf{r} = \mathbf{a} + t\mathbf{n}$.
- ☐ $(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$.
- ☐ $(\mathbf{r} - \mathbf{a}) \times \mathbf{n} = \mathbf{0}$.

2. The surface described by

$$x^2 + y^2 = z^2 + 4$$

is a

- ☐ sphere.
- ☐ paraboloid.
- ☐ hyperboloid.

3.

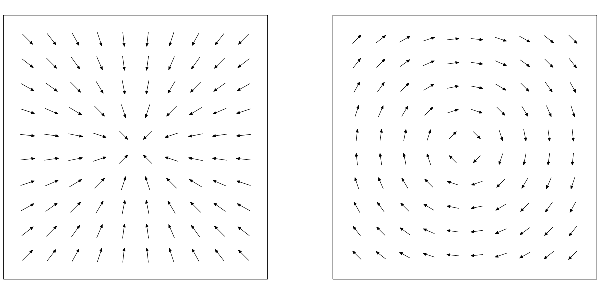

Figure for question 3.

In the diagrams above

- ☐ the left hand vector field has non-zero divergence and the right non-zero curl.
- ☐ the left hand vector field has non-zero curl and the right non-zero divergence.
- ☐ neither of the above.

4. $\mathbf{F} = (2xz - y^2)\mathbf{i} + y^2\mathbf{j} + (zx^2 + xy)\mathbf{k}$ is a vector field. Its divergence $\nabla.\mathbf{F}$ is

- ☐ $2z\mathbf{i} + 2y\mathbf{j} + x^2\mathbf{k}$.
- ☐ $x^2 + 2y + 2z$.
- ☐ $x^2 + 3x + y + 2z + 2xz$

5. Only one of the following index notation expressions or equations obeys the rules for such expressions. Which?

- ☐ $a_p b_m k_l m_l c_m d_p b_m$
- ☐ $\epsilon_{mjk} b_k (c_p d_p) a_j = a_k$
- ☐ $a_m b_m = 5c_p d_p$

Continued overleaf

6. $V = xy^2 + 3xz$. The value of ∇V at the point $(2, -1, 1)$ is

- ☐ $4\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$.
- ☐ $2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$.
- ☐ 6
- ☐ 4

7. The curl of the vector $(x,\ y - z^2,\ yz)$ is

- ☐ $\mathbf{i} + \mathbf{j} + y\mathbf{k}$.
- ☐ $-z\mathbf{i}$.
- ☐ $2\mathbf{i} + (1 + y)\mathbf{j} + (1 - 2z)\mathbf{k}$.
- ☐ $3z\mathbf{i}$.

8. If $\mathbf{F} = y\mathbf{i} - 3z\mathbf{j} + 2x\mathbf{k}$, the integral $\int \mathbf{F}.\mathbf{dr}$ along the curve $\mathbf{r} = \cos t\,\mathbf{i} + 2\sin t\,\mathbf{j}$ from $(1,\ 0,\ 0)$ to $(-1,\ 0,\ 0)$ is

- ☐ π.
- ☐ 0.
- ☐ $-\pi$.
- ☐ -2π

9. $\mathcal{S}$ is the sphere of radius a centred at the origin, bounding a volume $\mathcal{V}$. $\mathcal{H}$ is the hemisphere of $\mathcal{S}$ in $z \geq 0$. The area element on both is taken so that $\mathrm{d}\mathbf{S}$ has a positive $\mathbf{k}$ component if $z > 0$. $\mathcal{C}$ is the circular curve defined by $\mathbf{r} = a\sin t\mathbf{i} + a\cos t\mathbf{j}$, enclosing the disk $\mathcal{D}$. $\mathbf{F}$ is a (suitably differentiable) vector field. One of the following is false. Which?

- ☐ $\int_{\mathcal{V}} \nabla \cdot \mathbf{F}\mathrm{d}V = \int_{\mathcal{S}} \mathbf{F}.\mathrm{d}\mathbf{S}$.
- ☐ $\int_{\mathcal{H}} \nabla \times \mathbf{F}.\mathrm{d}\mathbf{S} = \int_{\mathcal{C}} \mathbf{F}.\mathrm{d}\mathbf{r}$.
- ☐ $\int_{\mathcal{H}} \nabla \times \mathbf{F}.\mathrm{d}\mathbf{S} = \int_{\mathcal{D}} \nabla \times \mathbf{F}.\mathbf{k}\,\mathrm{d}x\,\mathrm{d}y$.
- ☐ If $\nabla \times \mathbf{F} = \mathbf{0}$, then $\int_{\mathcal{C}} \mathbf{F}.\mathrm{d}\mathbf{r} = 0$ and $\mathbf{F}$ is conservative.

10. Given that

$$\epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl} \ ,$$

which of the following is not equal to $\epsilon_{mki}\epsilon_{ilm}$?

- ☐ $2\delta_{kl}$.
- ☐ $\delta_{ij}\epsilon_{mkj}\epsilon_{mil}$
- ☐ $-2\delta_{kl}$.
- ☐ $-\epsilon_{mik}\epsilon_{mil}$

End of test