So the ratio of the two energies is **marks**

$$\frac{E_{\text{trans}}}{E_{\text{pot}}} = -\frac{h^2}{8\pi^2 m_e n^2 a_0^2} \times \frac{4\pi\varepsilon_0 n^2 a_0}{e^2} = -\frac{h^2\varepsilon_0}{2\pi m_e a_0 e^2}$$ 2

$$= -\frac{(6.63 \times 10^{-34}\,\text{J s})^2 \times 8.85 \times 10^{-12}\,\text{C}^2\,\text{N}^{-1}\,\text{m}^{-2}}{2\pi \times 9.11 \times 10^{-31}\,\text{kg} \times 5.29 \times 10^{-11}\,\text{m} \times (1.60 \times 10^{-19}\,\text{C})^2}$$

$$= -0.50.$$ 3

Marks for working: *11*

Checking

- It is reasonable that E_{trans} should increase with increasing $|-E_{\text{pot}}|$, from the viewpoint of Schrödinger's equation. A very large negative potential energy leads to an electron that is confined to a very small region. By the uncertainty principle, one would expect such an electron to have a high kinetic energy.
- Treating the electron as a quantum-mechanical particle in a one-dimensional infinite square well whose width D is equal to the circumference $2\pi n^2 a_0$ of the nth Bohr orbit gives a kinetic energy of

$$\frac{n^2h^2}{8m_eD^2} = \frac{n^2h^2}{8m_e \times 4\pi^2 n^4 a_0^2} = \frac{h^2}{32\pi^2 m_e n^2 a_0^2}$$

which has the same order of magnitude (to within a factor of 4) as our answer.

(*Note*: Only one appropriate check is required for full marks.) 2

Marks for checking: 2

Total for Q20: **16**

$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$ $L = \frac{nh}{2\pi} = rmv$

$r = \frac{e^2}{4\pi\varepsilon_0 mv^2}$ $r = \frac{nh}{2\pi mv}$

$\frac{e^2}{\cancel{2\pi\varepsilon_0 mv^2}} = \frac{nh}{\cancel{2\pi mv}}$

$v = \frac{e^2}{2nh\varepsilon_0}$ n = 4, r = 8.43E-10

n = 5 r = 1

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