Then, for the proton,

$$(a_\mathrm{p})_x = e\mathcal{E}_x/m_\mathrm{p}.$$

Using $$(s_\mathrm{p})_x = (u_\mathrm{p})_x t + \tfrac{1}{2}(a_\mathrm{p})_x t^2,$$

the proton's displacement when it passes the electron is

$$d = \tfrac{1}{2}(a_\mathrm{p})_x t^2, \quad \text{since } (u_\mathrm{p})_x = 0.$$ 1

For the electron,

$$(a_\mathrm{e})_x = -e\mathcal{E}_x/m_\mathrm{e},$$ 2

and its displacement as it passes the proton is

$$d - S = \tfrac{1}{2}(a_\mathrm{e})_x t^2.$$ 1

Then $$\frac{d-S}{d} = \frac{\frac{1}{2}t^2(-e\mathcal{E}_x/m_\mathrm{e})}{\frac{1}{2}t^2(e\mathcal{E}_x/m_\mathrm{p})} = -\frac{m_\mathrm{p}}{m_\mathrm{e}}. \qquad \text{(i)}$$ 1

Therefore $$1 - \frac{S}{d} = -\frac{1.67 \times 10^{-27}\,\mathrm{kg}}{9.11 \times 10^{-31}\,\mathrm{kg}} = -1.83 \times 10^3.$$ 1

Hence $$d = \frac{3.7 \times 10^{-2}\,\mathrm{m}}{1.83 \times 10^3} = 2 \times 10^{-5}\,\mathrm{m},$$ 1

i.e. the particles cross just 20 μm from the positive plate.

Marks for working: *10*

Checking

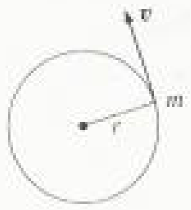
Equation (i) shows that if the two particles were of equal mass (and magnitude of charge), they would cross at $d = S/2$, exactly halfway across the gap, which is what one would expect.

Marks for checking: *1*

Total for Q19: 16

Question 20

Preparation



1

Figure 6

Relevant equations and principles:

$$L = mvr = n\hbar \quad \text{with } \hbar = \frac{h}{2\pi}$$

$$E_\mathrm{trans} = \tfrac{1}{2}mv^2 \quad \text{and} \quad E_\mathrm{pot} = -\frac{e^2}{4\pi\varepsilon_0 r_n}$$ 2

Marks for preparation: *3*

Working

In an orbit with quantum number n, the speed of the electron is

$$v = \frac{n\hbar}{m_\mathrm{e} r_n} = \frac{n\hbar}{m_\mathrm{e} n^2 a_0} = \frac{\hbar}{m_\mathrm{e} n a_0}.$$ 2

The kinetic energy of the electron is

$$E_\mathrm{trans} = \tfrac{1}{2}m_\mathrm{e}v^2 = \tfrac{1}{2}m_\mathrm{e}\left(\frac{\hbar}{m_\mathrm{e} n a_0}\right)^2 = \frac{h^2}{8\pi^2 m_\mathrm{e} n^2 a_0^2}.$$ 2

The potential energy of the electron is

$$E_\mathrm{pot} = -\frac{e^2}{4\pi\varepsilon_0 n^2 a_0}.$$ 2