Q.2–2 This question concerns a beam of particles of mass m and sharply defined energy $9\hbar^2k^2/2m$, where k is a positive real constant, incident from the left on a one-dimensional potential step given by the potential energy function

$$V(x) = \begin{cases} 0 & x \leqslant 0 \\ \dfrac{5\hbar^2k^2}{2m} & x > 0 \end{cases}$$

The solution to the time-independent Schrödinger equation is of the form

$$\psi(x) = \begin{cases} A\exp(\mathrm{i}k_1x) + B\exp(-\mathrm{i}k_1x) & x \leqslant 0 \\ C\exp(\mathrm{i}k_2x) + D\exp(-\mathrm{i}k_2x) & x > 0 \end{cases}$$

where k_1 and k_2 are positive real constants.

(i) By substituting in the time-independent Schrödinger equation, derive expressions for k_1 and k_2 in terms of k.

(ii) State which of the coefficients A, B, C and D is zero, explaining the reasons for your answer.

(iii) State the general conditions on the wave function $\psi(x)$ which give two relations between the non-zero coefficients. Hence express all the non-zero coefficients in terms of A.

(iv) Show that the reflection coefficient R is $\frac{1}{25}$.

(v) State a general relation between the reflection and transmission coefficients and hence find the transmission coefficient T for the potential step.

(vi) Compare the quantum values for the reflection and transmission coefficients with their values according to *classical* mechanics.
