PART II

Instructions for Part II

(i) *You should attempt* **THREE QUESTIONS ONLY** *from this part.*

(ii) *You may answer questions in any order, writing your answers in the answer book(s) provided.*

(iii) *All questions in this part carry equal marks.*

(iv) *All working* **MUST** *be shown. If you are sure that there is something that you do not wish the examiner to mark, cross it out. It will then be ignored totally.*

(v) *You are advised to spend about* **90 minutes** *on Part II, which carries 51% of the total marks for the examination.*

Q.2–1

(i) Write down Schrödinger's time-independent equation for a particle of mass m moving in an infinite square well potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 \leqslant x \leqslant L \\ \infty & x > L \end{cases}.$$

Verify that the wave function

$$\psi(x) = N \sin\left[(2mE)^{1/2} x/\hbar\right]$$

is a solution over the range $0 \leqslant x \leqslant L$, where E is the energy of the particle and N is a positive real constant.

(ii) State the boundary conditions that the wave function $\psi(x)$ must satisfy and show that the values of E which are compatible with these boundary conditions are

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2} \qquad (n = 1, 2, 3, \ldots).$$

(iii) State the normalization condition for the wave function $\psi_n(x)$ and find the value of the constant N which is compatible with this condition.

(iv) Determine the expectation values $\langle x \rangle$ and $\langle x^2 \rangle$ for the state described by the wave function $\psi_n(x)$ and hence show that the uncertainty $\Delta(x)$ for a measurement of position is

$$\Delta(x) = \frac{L}{2\sqrt{3}}\left(1 - \frac{6}{n^2\pi^2}\right)^{1/2}.$$

You may find the following integrals useful:

$$\int \sin^2 kx\, dx = \tfrac{1}{2}x - \frac{1}{4k}\sin 2kx.$$

$$\int x\sin^2 kx\, dx = \tfrac{1}{4}x^2 - \frac{x}{4k}\sin 2kx - \frac{1}{8k^2}\cos 2kx.$$

$$\int x^2\sin^2 kx\, dx = \tfrac{1}{6}x^3 - \left(\frac{x^2}{4k} - \frac{1}{8k^3}\right)\sin 2kx - \frac{x}{4k^2}\cos 2kx.$$
