Q.1–6 In this question, $\hat{\mathrm{A}}$ and $\hat{\mathrm{B}}$ are non-commuting operators representing different observables A and B of a certain system. Ψ and Φ are normalized vectors satisfying the eigenvalue equations

$$\hat{\mathrm{A}}\Psi = a\Psi \quad \text{and} \quad \hat{\mathrm{B}}\Phi = b\Phi,$$

where a and b are discrete non-degenerate eigenvalues.

(i) Select the **ONE** option from the key that gives the probability that A will be found to have the value a immediately after B has been found to have the value b.

(ii) Select the **ONE** option from the key that gives the expectation value for a measurement of A immediately after B has been found to have the value b.

KEY for Q.1–6 (i) and (ii)

A (Ψ, Φ)

B (Φ, Ψ)

C $(\Psi, \Phi)^2$

D $(\Phi, \Psi)^2$

E $|(\Psi, \Phi)|^2$

F $(\Psi, \hat{\mathrm{A}}\Phi)$

G $(\Phi, \hat{\mathrm{A}}\Phi)$

H $(\Psi, \hat{\mathrm{B}}\Psi)$

I $(\Phi, \hat{\mathrm{B}}\Psi)$

Q.1–7 This question concerns energy eigenstates of the Coulomb model of the hydrogen atom, represented by wave functions of the form

$$\psi_{n,l,m}(r,\theta,\phi) = R_{n,l}(r)\,Y_{l,m}(\theta,\phi),$$

where r, θ, ϕ are spherical polar coordinates.

(i) Choose the **TWO** options from the key that specify combinations of quantum numbers that are *not* allowed for an energy eigenstate.

(ii) Choose the **ONE** option from the key that represents a state bound by an energy $\frac{1}{4}E_{\mathrm{R}}$, where $E_{\mathrm{R}} = 13.61\,\mathrm{eV}$ is the Rydberg energy.

(iii) Choose the **ONE** option from the key that represents a state whose probability distribution has precisely two concentric spherical nodal surfaces.

KEY for Q.1–7 (i) to (iii)

A $\psi_{1,0,0}$

B $\psi_{2,1,0}$

C $\psi_{3,1,1}$

D $\psi_{3,1,2}$

E $\psi_{3,3,2}$

F $\psi_{4,2,-1}$

G $\psi_{5,2,-1}$

H $\psi_{5,4,-4}$
