SM355
Specimen Exam Solutions

PART I

		Marks
Q.1–1	(i) D (ii) E (iii) F	3
Q.1–2	✓ × × ✓ × ✓ ✓ ✓	3
Q.1–3	(i) ADEFG (ii) DF (iii) BC	4
Q.1–4	(i) F (ii) A (iii) D	3
Q.1–5	(i) C (ii) G (iii) B (iv) D	4
Q.1–6	(i) A (ii) J (iii) H (iv) F (v) G	4
Q.1–7	(i) I (ii) F (iii) G (iv) C (v) B (vi) C (vii) A (viii) A (ix) D	5
Q.1–8	(i) B (ii) A (iii) D	3

PART II

Q.2–1 (i) $\int_{-\Delta}^{\Delta} N^2\,dx = 1 \qquad \therefore N = (2\Delta)^{-1/2}$

(ii) $\langle x\rangle = \dfrac{1}{2\Delta}\int_{-\Delta}^{\Delta} x\,dx = 0$

(iii) $P(E_n) = |(\psi_n,\phi)|^2$

$$(\psi_n,\phi) = \int_{-a/2}^{a/2} \psi_n^*(x)\phi(x)\,dx$$

$\phi(x)$ has odd parity while $\psi_n^*(x)$ has even parity for $n = 1, 3\ldots$ $\therefore P(E_n) = 0$ for $n = 1, 3\ldots$. For $n = 2, 4\ldots$

$$\begin{aligned}(\psi_n,\phi) &= \left(\frac{2}{a}\right)^{1/2}\left(\frac{1}{2\Delta}\right)^{1/2} 2\int_0^{\Delta} \sin\left(\frac{n\pi x}{a}\right)dx \\ &= \frac{2}{(a\Delta)^{1/2}}\left[-\frac{a}{n\pi}\cos\left(\frac{n\pi x}{a}\right)\right]_0^{\Delta} \\ &= \frac{2}{n\pi}\left(\frac{a}{\Delta}\right)^{1/2}\left[1-\cos\left(\frac{n\pi\Delta}{a}\right)\right] \\ &= \frac{4}{n\pi}\left(\frac{a}{\Delta}\right)^{1/2}\sin^2\left(\frac{n\pi\Delta}{2a}\right)\end{aligned}$$

$$\therefore P(E_n) = \frac{16a}{n^2\pi^2\Delta}\sin^4\left(\frac{n\pi\Delta}{2a}\right) \quad n = 2, 4\ldots$$

Q.2–2 (i) For stationary state

$$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$$
$$\Psi^*(x,t) = \psi^*(x)e^{iEt/\hbar}$$

$$\begin{aligned}J &= -\frac{i\hbar}{2m}\left\{\psi^*(x)e^{iEt/\hbar}\frac{\partial\psi}{\partial x}(x)e^{-iEt/\hbar} - \psi(x)e^{-iEt/\hbar}\right. \\ &\quad\left.\times\frac{\partial\psi^*}{\partial x}(x)e^{iEt/\hbar}\right\} \\ &= -\frac{i\hbar}{2m}\left\{\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right\}.\end{aligned}$$

(ii) (a) continuity of $\psi : A + B = C$
continuity of $\psi' : ikA - ikB = -qC$

(b) Eliminate $C : ik(A - B) = -q(A + B)$
$\therefore A(ik + q) = B(ik - q)$

reflection coefficient:

$$\left|\frac{B}{A}\right|^2 = \left(\frac{ik+q}{ik-q}\right)\left(\frac{-ik+q}{-ik-q}\right) = 1.$$

All particles in the beam are reflected.

(c) For $x > 0$,

$$J = -\frac{i\hbar}{2m}\{C^*e^{-qx}C(-q)e^{-qx} - Ce^{-qx}C^* \times(-q)e^{-qx}\} = 0$$