Because Φ_n^E are normalized, $\sum_{m=0}^{\infty} |b_{nm}|^2 = 1$.

To test $E = 0$, we form the operator $\widehat{\mathbf{H}}_{\text{sho}} - 0\widehat{\mathbf{1}} = \widehat{\mathbf{H}}_{\text{sho}}$

$$\therefore \eta_n^0 = \widehat{\mathbf{H}}_{\text{sho}}\Phi_n^E = \sum_{m=0}^{\infty} b_{nm}\widehat{\mathbf{H}}_{\text{sho}}\psi_m = \sum_{m=0}^{\infty} b_{nm}E_m\psi_m$$

$$\therefore c_n^0 = \left(\sum_{m=0}^{\infty} b_{nm}E_m\psi_m, \sum_{r=0}^{\infty} b_{nr}E_r\psi_r\right)$$

$$\therefore c_m^0 = \sum_{m=1}^{\infty} |b_{nm}|^2|E_m|^2.$$

Smallest possible value of $E_m = \frac{1}{2}\hbar\omega_0$

$$\therefore c_n^0 \geq \frac{1}{2}\hbar\omega_0 \sum_{m=1}^{\infty} |b_{nm}|^2$$

$$\therefore c_m^0 \geq \frac{1}{2}\hbar\omega_0 \text{ for any } n.$$

$\therefore 0$ is not in $\mathrm{Sp}(\widehat{\mathbf{H}}_{\text{sho}})$.

Q.2–6 (i)

$$\mathbf{S}_x = \frac{\hbar}{2}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$\mathbf{S}_z = \frac{\hbar}{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$$

$$\mathbf{S}_x\mathbf{S}_z = \frac{\hbar^2}{4}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar^2}{4}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$$

$$\mathbf{S}_z\mathbf{S}_x = \frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = \frac{\hbar^2}{4}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$$

$$\mathbf{S}_x\mathbf{S}_z - \mathbf{S}_z\mathbf{S}_x = \frac{\hbar^2}{2}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix} \neq \mathbf{0}$$

$\therefore \mathbf{S}_x$ and $\mathbf{S}_z$ do not commute.

(ii)

$$\mathbf{S}_x^2 = \mathbf{S}_x\mathbf{S}_x = \frac{\hbar^2}{4}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} = \frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\mathbf{S}_z^2 = \frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix} = \frac{\hbar^2}{4}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\Delta(S_x) = (\langle S_x^2\rangle - \langle S_x\rangle^2)^{1/2}$$

$$\langle S_x^2\rangle = \frac{\hbar^2}{4}\left(\frac{1}{2}\right)[1 \quad 1]\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{\hbar^2}{8}[1 \quad 1]\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{\hbar^2}{4}$$

$$\langle S_x\rangle = \frac{\hbar}{2}\left(\frac{1}{2}\right)[1 \quad 1]\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{\hbar}{4}[1 \quad 1]\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{\hbar}{2}.$$

$\therefore \Delta(S_x) = 0$

$$\langle S_z^2\rangle = \langle S_x^2\rangle = \frac{\hbar^2}{4}$$

$$\langle S_z\rangle = \frac{\hbar}{2}\left(\frac{1}{2}\right)[1 \quad 1]\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix} = \frac{\hbar}{4}[1 \quad 1]\begin{bmatrix}1\\ -1\end{bmatrix} = 0.$$

$$\therefore \Delta(S_z) = (\langle S_z^2\rangle - \langle S_z\rangle^2)^{1/2} = \left(\frac{\hbar^2}{4}\right)^{1/2} = \frac{\hbar}{2}.$$

(iii) Left hand side is $0 \times \frac{\hbar}{2} = 0$

Right hand side is $\frac{1}{2}\left|\frac{1}{2}\frac{\hbar^2}{2}[1 \quad 1]\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\begin{bmatrix}1\\ 1\end{bmatrix}\right|^2 = \frac{\hbar^2}{8}[1 \quad 1]\begin{bmatrix}-1\\ 1\end{bmatrix} = 0$

$\therefore$ the generalized uncertainty principle is satisfied.

PART III

Q.3–1 *A good answer would include the following points:*

(i) Oven to produce a beam of neutral Cs or Ag (unpaired electron) atoms.

(ii) Magnet with shaped pole pieces to produce

$\left\{\begin{array}{l}\text{a magnetic field and}\\ \text{a magnetic field gradient}\end{array}\right\}$ perpendicular to beams

(iii) Beam deflection proportional to component of magnetic moment: $F = \mu_z \partial B_z/\partial z$ where F is the force on the beam, μ_z is the component of the magnetic dipole moment of each atom in the z-direction and B_z is the component of the magnetic field in the z-direction.

(iv) Deflected beam detected by hot wire detector.

(v) Diagrams:

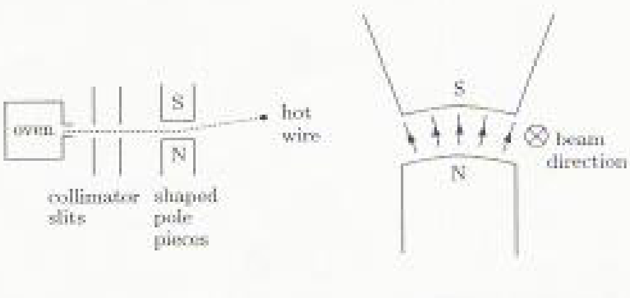

Figure 1