MST121 – 1999 Solutions

Qn.1 (a) 3, 2.9, 2.8, 2.7
(b) Arithmetic Progression
$x_0 = 3 \;\; x_{n+1} = x_n - 0.1$ n=0,1,2,..
(c) yes, after 30 secs.
x_n becomes indefinitely large and negative.

Qn.2 (a) $\frac{2}{27}, \frac{2}{81}$
(b) Geometric Progression
$x_n = \frac{6}{3^n}$ n=0,1,2, ...
(c) it tends to 0 in the long run.
x_n oscillates towards 0.

Qn.3(a) $(x-3)^2+(y-1)^2=16$, radius=4, centre is (3,1), so (b) dist from O is $\sqrt{10}$
(c) $\sqrt{10} < \sqrt{16}$ so O is inside circle. Also centre (3,1) is in first quadrant so C is the correct diagram.

Qn.4 (a) 4%
(b) $f : \mathrm{N}^+ \to \Re$
$n \mapsto 2000(1.04)^n$
*** (n.b. formal defns. no longer in syllabus)
(c) Solve $A=2000(1.04)^n$ for n
$n = \ln(A/2000) / \ln(1.04)$
$f^{-1}(3000) = \frac{\log 1.5}{\log 1.04} = 10.34$ so it requires 11 yrs. to exceed £3,000.

Qn.5 (a) *** no longer in syllabus

mem1	mem2	mem3
x		
x	x-1	
x	$(x-1)^2$	
x	$(x-1)^2$	x^2
x	$(x-1)^2$	x^2-4
x	$(x-1)^2/(x^2-4)$	x^2-4

(b) x=+2 and x=-2

Qn.6 (a)
(i) 0.75+0.25-0.25-0.75=0
(ii)successive terms become more and more negative, so sum becomes indefinitely large and negative.

(b)(i) $\frac{1.25(-0.5)(1-0.5^4)}{1.5} = -\frac{5}{12} \times \frac{15}{16} = -0.391$

(ii) $(1-(-0.5)^n)$ tends to 1, so sum tends to –5/12=0.4167

Qn.7 (a) It becomes a 2-cycle alternating between 2 values, one above 3,000 the other below – see B1 p.40.
(b) Rewrite eqn. as
$P_{i+1} - P_i = P_i(0.7 - 0.00035P_i)$ so
$P_{i+1} - P_i = 0$ means $P_i = E$
that is $E = \frac{0.7}{0.00035} = 2{,}000$

Qn.8 (a)(i) P+Q= $\begin{pmatrix} 2 & 1 \\ -2 & 0 \end{pmatrix}$,
QP = $\begin{pmatrix} 5 & 27 \\ -3 & -15 \end{pmatrix}$
(b) M= $\begin{pmatrix} 2 & -5 \\ -1 & 4 \end{pmatrix}$ k= $\begin{pmatrix} -9 \\ 6 \end{pmatrix}$
$M^{-1}= \begin{pmatrix} 4/3 & 5/3 \\ 1/3 & 2/3 \end{pmatrix}$ soln.= $M^{-1}k= \begin{pmatrix} -2 \\ 1 \end{pmatrix}$

Qn.9 (a) v = 30 - 10t
(b) Max ht. when v=dh/dt=0 so t=3.
$d^2h/dt^2 = -10 < 0$ so max A at t=3 is
35+90-45=80
(c) h=0 when $5(7+6t-t^2)=0$
i.e. (7-t)(1+t)=0 so t=7 secs.

Qn.10 (a) $f'(t)=-3e^{-t}-12t^2$
const multiple and sum rules
(b) $2\ln x - \cos(5x) + c$;const multiple and sum rules

Qn.11 (a)(0,0) corresponding to $(1-e^x)=0$ and (2,0) corr to (x-2)=0.
If 0<x<2, x-2<0 and $e^x>1$ so
$(1-e^x)>0$ hence $(x-2)(1-e^x)>0$

(b) $[\tfrac{1}{2}x^2-2x+(3-x)e^x]_0^2$
$=(2-4+e^2)-(0-0+3)= e^2-5=2.389$

Qn.12 (a) diminishing exponential curve starting at (0,35) and asymptotic to θ=20
(b) A=20, B=15
(c) 25=20+15.exp(-0.43t)
ln(1/3)=-0.43t
t=ln(3)/0.43 = 2.56 hrs.

Qn.13 (a) 4=1+3=3+1=2+2 so proby = 3/16.
(b) 4 doubles so proby.= ¼
(c) py(not double)=¾, so
py(8 not-doubles)=$(¾)^8$ = 0.10
(d) 1- py(8 not-doubles)= 0.90
(e) mean no of times = 1/py=4

Qn.14 (a) median:242, LQ : 224
UQ : 251, Range:70, IQR : 27
(c) Patterns are broadly similar but there is more variation at the extremes for males.

Qn.15 $3430 \pm 1.96 \times \frac{487}{20}$ =(3382,3478)

Qn.16 (a) 83+0.54x160=169.4 cm.
(b)10x0.57 (see eqn.(1))=5.7cm.

Qn.17 (a) (i) A is (-1,0) B is (5,12)
(ii) x=2t-1; 2t=x+1; so
$y=2t^2-4(2t)=(x+1)^2-4(x+1)$
$= x^2-2x-3$
(b)(i)$y=(x-1)^2-4$ Min. when
$(x-1)^2=0$, ie x=1 so y=-4
(ii) $x^2-2x-3=(x-3)(x+1)$ so curve cuts x axis at (3,0) and (-1,0).
y(0)=-3 so curve cuts y-axis at (0,-3).
Curve is parabola with minimum and vertex at (1,-4)
(c) grad of AB =12/(5+1)=2 and it goes through (-1,0) so eqn is y=2(x+1)

Qn.18 (a) (i)
$y_0 = 48.42, E_0 = 8.99, total = 57.41$
propn of elderly= $\frac{8.99}{57.41} = 0.157$
$y_{n+1} = 0.9991 y_n, E_{n+1} = 0.0143 y_n + 0.9325 E_n$
0.9991 is ratio of present young popn to last years young popn;
0 means no entrants from young to elderly;
0.0143 is propn of young who become elderly each year;
0.9325 is propn of elderly who survive each year.
(b) see graphs on p.17 – they would level out even more until they were nearly horizontal.
(c) (i) size of total popn will continue to fall indefinitely;
(ii) propn of elderly will rise towards an asymptotic level.

Qn.19 (a) $\int_0^1 3x^2\,dx = \left[x^3\right]_0^1 = 1$
(b)$y=3x^2$, y'=6x so gradient at T = 6t
Eqn of tangent: $y-3t^2=6t(x-t)$
$y=6tx-3t^2$

(c) y=0 => x=½t so PR=(1-½t)
x=1 =>$y=6tx-3t^2=3t(2-t)$
area of triangle= ½ base x ht
$= ¾t(2-t)^2$

(d) $A=¾t(4-4t+t^2)=3t-3t^2+¾t^3$
$dA/dt= 3-6t+9/4t^2 = 0$ in (0,1)
when $9t^2-24t+12=0$
ie $3(3t^2-8t+4)=3(3t-2)(t-2)=0$
ie when t=2/3 for which the value of $A=¾(2/3)(4/3)^2=8/9$
A at t=0 is 0, A at t-1 is ¾, so 8/9 is maximum value.
Check : area found in (a)=1 >8/9

Qn.20 (a) (i) The distribution is skewed to the right
(ii) Normal curve shaded from left up to x=500.
(b) (i)
$502.4 \pm 1.96 \times \frac{9.79}{\sqrt{50}} = (499.7, 505.1)$
so ans = (500,505) to 3 s.figs
(ii) There is no overwhelming evidence to suggest that the mean weight is below 500gm., on the other hand the evidence is a bit marginal.
(iii) (1) 200 : sample standard devn varies with the reciprocal of the sq root of sample size.
(2) no – because each sample has its own standard devn, and this statistic is subject to its own variability.