MST121 – 1998

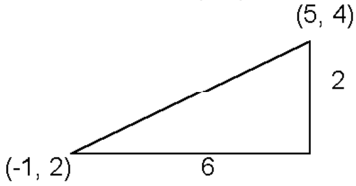
1(a) 7, 6.8, 6.6, 6.4   (b) Arithmetic $x_1 = 7$, $x_{n+1} = x_n - 0.2$, $n = 1, 2, 3, \ldots$

2(a) 1, 1/2, 1/4, 1/8   (b) Geometric $x_n = (1/2)^n$, $n = 0, 1, 2, \ldots$

3(a) Start is (-1, 2) After 2 secs is (5, 4)

(b) $x = 3t - 1$ so $t = (1/3)x + 1/3$ Substitute this into $y = t + 2$ to give $y = (1/3)x + 7/3$ Straight line.

(c)



Distance $= \sqrt{(6^2 + 2^2)} = 6.32$

4(a) $y = 0$ gives $\frac{x(40-x)}{20} = 0$ ie. $x = 0$ or 40 Hence, 40 metres.

(b) $0 \le x \le 40$ Hence, $f : [0, 40] \to R^+$

$x \to \frac{x(40-x)}{20}$

(c) When $x = 38$, $y = 3.8$ which is greater than the 3m bar. Hence, kick successful.

(d) Parabola. Highest point at $x = 20$, giving $y = 20$ metres.

5(a)

Memory	Screen
" "	---
" "	"pan"
"n"	"pan"
"n"	"pa"
"n"	"pai"
"n"	"pain"
" "	"pain"

(b) screen := ADDLAST ('t', screen)

6(a) Tends to $\frac{0.9}{1-0.9} - 9$ (b) It diverges

7(a) $P_{i+1} = P_i[1 + 1.9(1 - \frac{P_i}{300})]$ Use $P_i = P_0 = 350$ to find $P_{i+1} = P_1 = 239.2$

(b) Change E to 500 and r to 1.5

(c) For r in the range $1 < r < 2$ P_i will converge on the equilibrium population of 300, with values alternating above and below E.

8(a)(i) $\begin{pmatrix} 3 & 0 \\ 2 & -4 \end{pmatrix}$ (ii) $\begin{pmatrix} 17 & -10 \\ -13 & 5 \end{pmatrix}$

(b)(i) $A = \begin{pmatrix} 1 & -3 \\ 2 & 5 \end{pmatrix}$ $b = \begin{pmatrix} 9 \\ -4 \end{pmatrix}$

(ii) $A^{-1} = 1/11\begin{pmatrix} 5 & 3 \\ -2 & 1 \end{pmatrix}$ $1/11\begin{pmatrix} 5 & 3 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 9 \\ -4 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ Hence, $x_1 = 3$, $x_2 = -2$

9(a) when $t = 0$, $x = 100[1 - \cos(\frac{\pi * 0}{60})] = 0$ when $t = 30$, $x = 100[1 - \cos(\frac{\pi * 30}{60})] = 100$

$\therefore$ jogger covers $100 - 0 = 100$ metres

(b) velocity $= dh/dt = -100[-\sin(\pi t/60)](\pi/60) = (5\pi/3)\sin(\pi t/60)$

Max when $\sin(\pi t/60) = 1$ ie. $t = 30$, giving $v = 5\pi/3$ ms^{-1}

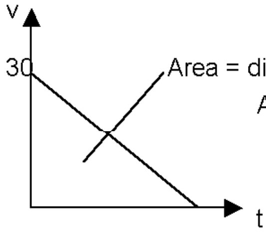
(c) acceleration $= d^2h/dt^2 = (\pi^2/36)\cos(\pi t/60)$

Max when $\cos(\pi t/60) = 1$ ie. $t = 0$, giving $a = \pi^2/36$ ms^{-2}

10(a) $f'(x) = 20t^4 - 6\cos(6t)$ Both use sum and constant multiple rule.

(b) $\int g(x)dx = -(1/2)e^{-2x} - 2x^{3/2} + c$

11(a)

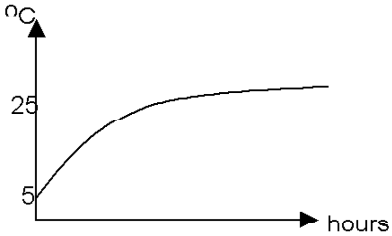


Area = distance = 90 = (1/2)t * 30 $\therefore$ t = 6

A 30 ms^{-1} change in 6 secs is a deceleration of 30/6 = 5 ms^{-2}

(b) From above, 6 secs

12(a)



(b) When t large, $\exp(-0.575t) \to 0$ so θ approaches A

Hence, A = 25

When t = 0, θ = A – B = 5 Hence, B = 20

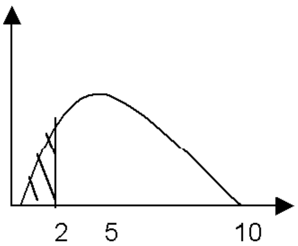
(c) $10 = 25 - 20\exp(-0.575t)$

$\Leftrightarrow \exp(-0.575t) = 0.75$

$\Leftrightarrow -0.575t = \ln(0.75)$

$\Leftrightarrow$ 0.5003 Temp at 10 degrees after 30 mins

13(a)



Shaded area is the required proportion.

(b)(i) $\mu = 3.5$, $\sigma = 0.3$

(ii) 3.5 ± 0.6 mins ie. [2.9, 4.1]

14(a) $1002 \pm 1.96 * 10/\sqrt{50}$ = [999.2, 1004.8] cm^3

(b) 1 litre is in the confidence interval. Hence, it is unlikely (though still possible) that the mean of all the bottles is less than one litre.

15(a) ESE = $\sqrt{(0.72^2/209 + 0.53^2/77)}$ = 0.07828 Z = (3.80 – 3.63)/ESE = 2.17

(b) Z > 2 so we reject the null hypothesis at the 95% confidence level in favour of the alternative hypothesis. We note that the sample for grassland has a higher mean than the sample for cultivated land, and so conclude that the clutch sizes are greater for grassland lapwings.

16(a)(1) Data is in the shape of a curve. (2) Most data points are below the line.

(b)(1) Draw a curve through the points. (2) Straight line parallel to that given, but lower down

17(a) $b_0 = 6000$, $b_n = 1.015\, b_{n-1} - 150$, n = 1, 2, 3, ...

(b) $b_n = (-4000)(1.015)^n + 10000$ b_{36} = 3163.44 which is the amount owed.

(c) $f : N \to R$

$n \to (-4000)(1.015)^n + 10000$

(d) $f^{-1} : R \to R$ $f^{-1}(0) = 61.54$ Hence, 62 payments are required.

$$x \to \frac{1}{\ln(1.015)} \cdot \ln\left(\frac{10000 - x}{4000}\right)$$

18(a) $J_{n+1} = 0.9 A_n$ $I_{n+1} = 0.7J_n + 0.5 I_n$ $A_{n+1} = 0.4 I_n + 0.8 A_n$

(b)(i) assumed it's constant and equal to 0.9 of the total population of adults.

(ii) constant and equal to 1 – 0.8 = 0.2 of pop of adults.

(iii) constant and equal to 1 – (0.5 + 0.4) = 0.1 of pop of immatures.

(c)(i) $\begin{pmatrix} 0 & 0 & 0.9 \\ 0.7 & 0.5 & 0 \\ 0 & 0.4 & 0.8 \end{pmatrix}\begin{pmatrix} 500 \\ 900 \\ 850 \end{pmatrix} = \begin{pmatrix} 765 \\ 800 \\ 1040 \end{pmatrix}$ (ii) $\begin{pmatrix} 1.6 & 1.44 & -1.8 \\ -2.24 & 0 & 2.52 \\ 1.12 & 0 & 0 \end{pmatrix}\begin{pmatrix} 500 \\ 900 \\ 850 \end{pmatrix} = \begin{pmatrix} 566 \\ 1022 \\ 560 \end{pmatrix}$

19(a)(i) $\int^{4}_{-4} (8 - (1/2)x^2)dx = [8x - (1/6)x^3]^{4}_{-4}$ = 128/3 metres

(ii) Smallest rectangle = 8*8 = 64 m^2 Largest triangle = 0.5*8*8 = 32 m^2

(b)(i) $A = 2w(8 - 0.5w^2) = 16w - w^3$

(ii) $dA/dw = 16 - 3w^2 = 0$ at stationary point. Hence, $w = \sqrt{(16/3)}$ But $d^2A/dw^2 = -6w$

Hence, $d^2A/dw^2 < 0$ for $w > 0$, and $\sqrt{(16/3)}$ is a max, w = 2.3094 m.

Substitute in $A = 16w - w^3$ to find A = 24.63 m^2

(iii) Proportion = 24.63/(128/3) = 0.577

20(a)(i) 1/10 * 1/10 = 1/100

(ii) 1 – 1/100 = 99/100, $(99/100)^{10} = 0.9044$, $1 - (99/100)^{10} = 0.0956$

(b)(i) 9/10 (ii) 1 * 9/10 * 8/10 = 0.72 (iii) 1 – P(all different) = 1 – 0.72 = 0.28

(iv) Tom chooses any number, then 1/10 that each of Dick and Harriet match.

Hence, 1 * 1/10 * 1/10 = 1/100 = 0.01

(v) P(at least 2) – P(all 3) = 0.28 – 0.01 = 0.27