MST121 – 1997

1(a) 3, 3.9, 4.8, 5.7 (b) Arithmetic $x_n = 2.1 + 0.9\,n$, $n = 1, 2, 3, \ldots$

2(a) 3, 2.7, 2.43, 2.187 (b) Geometric $x_1 = 3$, $x_{n+1} = 0.9\,x_n$, $n = 1, 2, 3, \ldots$

3(a) $D = 6$, $S = 20$
(b) Need $D \geq 0$, hence $P \leq 10$ Need $S \geq 0$, hence $P^2 \geq 6$
(c) $D = S$ gives $10 – P = 2P^2 – 12$, $2P^2 + P – 22 = 0$, $P = 1/4(-1 \pm \sqrt{(1 + 176)})$
Negative value is outside range. Hence, $P = 3.076$ Price is £3.

4(a)(i) $e^{-0.03*2} = e^{-0.06} = 94.2\%$ (ii) $e^{-0.03*20} = e^{-0.6} = 54.9\%$
(b) $f^{-1} : R^+ \rightarrow R^+$
$x \rightarrow -\ln x/0.03$
(c) $f^{-1}(0.6) = 17.03$ Hence, divergence for 1700 years.

5(a) x is a non-zero number in the range $-10 \leq x \leq 3$
(b) AND(x > 0, NOT (x = 3))

6(a) $E = 300$, $P_0 = 100$
(b) $P_0 = 100$, $P_{i+1} – P_i = 0.4\,P_i\,(1 – P_i/300)$
(c) Substitute $P_i = P_0 = 100$ and hence find $P_1 = 380/3$

7(a) Does not converge – alternates between 1 and 3.
(b) Converges on 2 since $(-0.5)^i$ becomes very small as i becomes large.
(c) Converges on 2 since $i^{-2}$ becomes very small as i becomes large.

8(a) $C + D = \begin{pmatrix} 9 & 0 \\ -2 & 5 \end{pmatrix}$ $DC = \begin{pmatrix} 22 & -2 \\ -12 & 10 \end{pmatrix}$

(b)(i) $A = \begin{pmatrix} 4 & -1 \\ 2 & 3 \end{pmatrix}$ $b = \begin{pmatrix} 11 \\ -5 \end{pmatrix}$

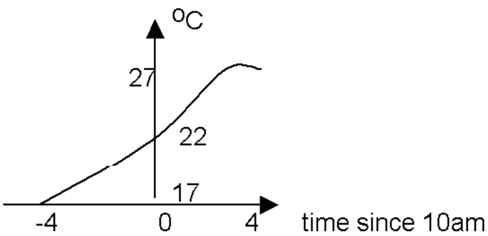
(ii) $A^{-1} = 1/14 \begin{pmatrix} 3 & 1 \\ -2 & 4 \end{pmatrix}$ $x = A^{-1} \begin{pmatrix} 11 \\ -5 \end{pmatrix} = 1/14 \begin{pmatrix} 28 \\ -42 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$ Hence, $x_1 = 2$, $x_2 = -3$

9(a) $v = h\,x^2$ $h = v/x^2 = 500/x^2$
(b) $ds/dx = 4x – 2000/x^2 = 0$ for stationary point. Hence, $4x – 2000/x^2 = 0$, $x = \sqrt[3]{500}$
$d^2s/dx^2 = 4 + 4000/x^3 > 0$ for $x = \sqrt[3]{500}$ Hence, this is a minimum.
(c) Substitute $x = \sqrt[3]{500}$ in $s = 2x^2 + 2000/x$ to find $s = 378.0\ cm^2$
(d) Substitute x from (b) above in $h = 500/x^2$ to find h = 7.94 cm.

10(a) $f'(x) = 8x^3 + 15\exp(3x)$ Both use sum and constant multiple rule.
(b) $\int g(t)dt = (3/4)\sin(4t) – \ln t + c$

11(a) Solve $x^2 – 4x + 3 = 0$ to give x = 1 and x = 3
(b) Area $= \int_1^3 (-x^2 + 4x – 3)dx = [-1/3\,x^3 + 2x^2 – 3x]_1^3 = 4/3$

12(a)



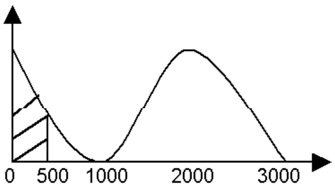

(b) A = 22, B = 5
(c) Period = 16 hours, hence $k = 2\pi/16 = \pi/8$

13(a) 1/6 * 1/6 = 1/36 (b) $(35/36)^{10} = 0.754$ (c) $1 - (35/36)^{10} = 0.246$

(d) Mean of geometric distribution is 1/p Hence, mean number of rolls = 36

14(a) Distribution has 2 peaks, normal distribution only has one.

(b)




(c) Standardize curve, so that the total area under curve is 1
Then shaded area represents proportion less than 500 hrs

15(a)(i) Mean = 1600, $\sigma = 12.5$ (ii) (1575, 1625)

(b)(i) (1564, 1596) (ii) Mean lifetime is significantly lower than before.

16 When x = 170, y = 174.4 Hence, sons are taller by 4.4 cm.

17(a) x = t Substitute this into equation for y to give y = 20 – x

(b) When y = 0, x = 20 Hence, $x_1 = 20$

(c) A = u * 0.5(20 + 20 – u) = 0.5u(40 – u) Hence, $A : [0, 20] \to R^+ \quad u \to 0.5u(40 - u)$

(d) A(20) = 200

(e) Solve 0.5u(40 – u) = 200/2 = 100 to find $u = 20 \pm \sqrt{200}$ But $u < 20$, hence u = 5.86

18(a)(i)

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          ÷
        /   \
      +       x
    /   \    / \
   x     +  2   y
  / \   / \
 y   y y   4
```

(ii)

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          +
        /   \
      ÷       ÷
     / \     / \
    +   2   2   y
   / \
  y   1
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(b)(i)

| Line | Mem One | Mem Two | Mem Three |
|---|---|---|---|
| 1 | y | --- | --- |
| 2 | y | $y^2$ | --- |
| 3 | y | $y^2$ | $y + y^2$ |
| 4 | y | $y^2$ | $y + y^2 + 4$ |
| 5 | 2y | $y^2$ | $y + y^2 + 4$ |
| 6 | 2y | $y^2$ | $(y + y^2 + 4)/2y$ |

(ii) The algorithm evaluates (i) directly.
However, (i) & (ii) are algebraically equivalent.

19(a) a(t) = v'(t) = cos(3t) $a(0) = 1\ ms^{-2}$

(b) $s(t) = \int v\,dt = -(1/9)\cos(3t) + c$ s(0) = 1/3 Hence, 1/3 = -1/9 + c so c = 4/9

$\therefore s(t) = -(1/9)\cos(3t) + 4/9 = (1/9)[4 - \cos(3t)]$

(c) Substitute cos(3t) = a into s = 1/9 [4 – cos(3t)] and rearrange to give the required a = -9s + 4

(d) Ball oscillates sinusoidally about a centre position 4/9 with range [1/3, 5/9]

20(a) Key values for A all slightly greater than for B. Overall range identical, inter-quartile range similar. Median for A is 0.2 kg greater – small compared to actual median value.

(b)(i) $H_0 : \mu_A = \mu_B$ where $\mu_A$ = mean weight of population of turkeys given feed A, and

$H_1 : \mu_A \neq \mu_B$ $\mu_B$ = mean weight of population of turkeys given feed B.

(ii) $$Z = \frac{7.0 - 6.7}{\sqrt{(0.88^2/72 + 0.96^2/80)}} = 2.01$$

(c) Since $Z > 1.96$, we reject the null hypothesis at the 95% confidence level in favour of the alternative hypothesis. We conclude the 2 populations means are not the same. As the sample mean for feed A is greater than that for feed B, we conclude that the mean weight of feed A turkeys is greater than the mean weight of feed B turkeys. However, 2.01 is very close to 1.96, so the evidence would not support rejection at a confidence level above 95%