MST121 – 2004 Solutions

Qn.1 (a) 6, 4.7, 3.4, 2.1
(b) Arithmetic Progression
x_n = 7.3 - 1.3n (n=1,2…)
(c) -44.7
(d) x_n becomes arbitrarily large and negative.

Qn.2 (a) (3,2)
(b) gradient of MN is –1/2 = grad AB
(c) gradient of AC = 2, gradAC x grad AB = -1
(d) y = 2x- 4, cuts x-axis at (2,0)

Qn.3 (a) $(x - 4)^2 + (y - 1)^2 = 25$ (b) (4,1), 5
(c) (6,-2) ; $(x - 6)^2 + (y + 2)^2 = 25$

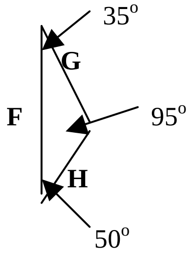
Qn.4 (a) b =4 (b) a=3
(c) add a mirror image in the x-axis
(d) increasing, one-to-one

Qn.5 (a) 5x150 + 3x75x151 = 34725
(b) -1/3 since n terms dominate for large n.
(c) since r = 2.3, long-term behaviour of P_n is a 2-cycle with one value above E = 500, and the other below 500.

Qn.6 (a) (i) AB does not exist
(ii) BC = $\begin{pmatrix} 8 & 4 \\ -6 & -3 \end{pmatrix}$
(b) (i) $\frac{1}{5}\begin{pmatrix} 1 & 3 \\ 1 & -2 \end{pmatrix}$
(ii) det(C) = 0, so C^{-1} does not exist

Qn.7 (a) –3**i** - 8**j** (b) $\sqrt{73}$ = 8.5 (to 1 d.p.), Direction is -110.6^0 to 1 d.p. or -1.9 radians. (Equivalently the answer could be given aa an anti-clockwise angle from the zero line of 249.4^0 or 4.4 radians.)

Qn.8 (a)



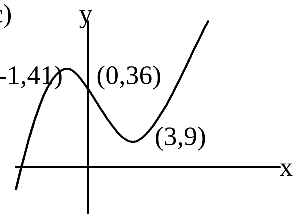
(b)

$$\frac{G}{\sin 50^0} = \frac{5}{\sin 85^0}$$

G = 3.84

Qn.9 (a)
$f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$
so stationary points exist at x = -1 and x = 3.
(b) $f''(x) = 6x - 6$ which is
-12 for x = -1, so f has a maximum
12 for x = 3, so f has a minimum

(c)



Qn.10 (a) $f'(x) = -\frac{4}{x^2} + 3\sin(6x)$
(b) $-3e^{-t/3} + 2t^{5t/2} + c$

Qn.11 (a) Solve 10 = a/2 + v_0
16 = 8a + 4 v_0
(b) ds/dt = at + v_0 . Max distance is attained when ds/dt=0, i.e. when t = -v_o/a = 3, at which time s = 18 m.

Qn.12 $0.75 = e^{-100k}$ so
k = -(ln(0.75))/100 = 0.0028768
For half-life solve $0.5 = e^{-0028768t}$
t = -(ln(0.5))/k = 241 years

Qn.13 (a) There are 3 ways of obtaining a 4 out of 36 possibilities, viz, (1,3), (2,2), (3,1), so probability =1/12.
(b) $(11/12)^5$ = 0.647 (c) 1 - 0.647 = 0.353
(d) mean number of rolls is 1/(1/12) = 12

Qn.14 (a) Mean = 20, s.d. = $\frac{9.5}{\sqrt{10}}$ = (20, 3.00)
(b) 95% Conf Int. is (14.1, 25.9)

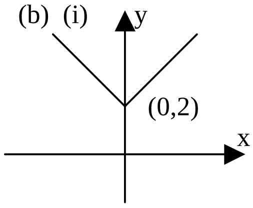
Qn.15 (a)) H_0: $m_0 = m_1$
H_1: $m_0 \neq m_1$
m_0 = mean consumption without additive
m_1 = mean consumption with additive

(b) The test statistic is 7.32 which is greater than 1.96, so reject the null hypothesis at the 5% significance level.
(c) It is very likely that the additive improves consumption.

Qn.16 (a) 0.187x170 - 7.19 = 24.6
(b) residual = data - fit
= 24 - 23.665 = 0.335
(c) Obtain the least squares fit line for the regression of x on y (as opposed to y on x).

Qn.17 (a) Scale *y* by a factor of 2, then translate 1 to the right and 5 down.

(b) (i)



(ii) $(x, y + 2)$

(c) y = |x| + 2 becomes y = -x + 2 to the left of the y-axis.

(d) (i) (3.27, 5.27) as given

(ii) Solve y = 2 - x and $y = 2(x - 1)^2 - 5$ to give (-1, 3)

Qn.18 (a) $J_{n+1} = 0.89\, J_n + 0.07\, A_n$

$A_{n+1} = 0.06\, J_n + 0.95\, A_n$

(b) (i) 0.07 (ii) 0.05

(c)

(i) $$\begin{pmatrix} Adults \\ Juv'niles \end{pmatrix} = \begin{pmatrix} 0.89 & 0.07 \\ 0.06 & 0.95 \end{pmatrix} \begin{pmatrix} 5.7 \\ 20.8 \end{pmatrix} = \begin{pmatrix} 6.529 \\ 20.102 \end{pmatrix}$$

Total = 26.631M

(ii) propn. of juveniles = 0.245

(d) (i) population size = 22.38M

(ii) propn. of juveniles = 0.407

Qn.19 (a) (i) use the chain rule

(ii) Product Rule gives

$$q'(x) = e^{-x^2/8} + xp'(x) = e^{-x^2/8}(1 - \tfrac{1}{4}x^2)$$

(iii) limits of integration are from -1 to 4.

$$\int_{-1}^{4} f(x)dx = \int_{-1}^{4} (4 + 3x - x^2)e^{-x^2/8}\, dx$$

$$= \int_{-1}^{4} 4q'(x)dx - 12\int_{-1}^{4} p'(x)dx$$

$$= \left[4q(x) - 12p(x)\right]_{-1}^{4}$$

$$= 4\{q(4) - 3p(4) + 3p(-1) - q(-1)\}$$

$$= 4\{e^{-2} + 4e^{-1/8}\}$$

= 14.66 to 2 d.p.

(i) $r'(x) = \dfrac{1 - \ln(ax)}{x^2}$

(ii) if $y = r(x)$, $\dfrac{1 - xy}{x^2} = \dfrac{1 - \ln(ax)}{x^2}$

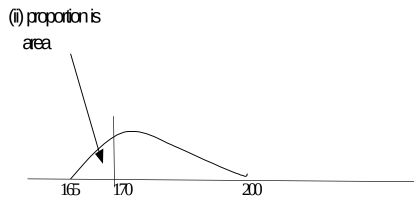
so general solution is $y = r(x)$.

y(1)=3 so 3 = ln(a),

so, using ln(ax)=ln(a) + ln(x), particular solution is $y(x) = \dfrac{\ln(3x)}{x}$

Qn.20 (a) Right-skew because it tails off to the right.

(b) (i) sketch is something like



(c) (i) $178.4 \pm 1.96 \times \dfrac{6.92}{\sqrt{60}}$

= (176.65, 180.15)

"95% confidence interval." means that 95% <u>all</u> such computed confidence intervals contain the true but unknown mean, whereas 5% do not. (see also p. 41 in Computer Book D),