MST121 – 2003 Solutions

Qn.1 (a) 20.4, 24.48, 29.376
(b) Geometric Progression
$x_n = 17(1.2)^{n-1}$ n=1,2...
(c) 128,923
(d) x_n becomes indefinitely large and positive.

Qn.2 (a) (3,-3)
(b) gradient of DE is -3, eqn. is y+3x=6
(c) gradient of BC is also –3
(d) $y = \frac{1}{3}x$

Qn.3 (a) $(x+1)^2+(y-3)^2=45$
(b) centre is (-1,3) , radius = $\sqrt{45}$

Qn.4
(a) Reflect graph in x-axis
(b) Draw a parallel line 2 units higher
(c) (i) f neither, g increasing
(ii) f is many-one and so does not have an inverse, e.g. $f(1) = f(-1) = 1$.
g has inverse $g^{-1}(x) = ½(x+5)$.

Qn.5 (a) -2, because for large n the n^2 terms dominate.
(b) ¼ because $4(0.8)^n$ tends to zero for large n.

Qn.6 (a) $\frac{20}{17}\begin{pmatrix} 0.95 & -0.1 \\ -0.05 & 0.9 \end{pmatrix}$
(b) 1981: (4.18, 6.985) 1989: (5.54,6.31)

Qn.7 b = $\sqrt{49+256-224\cos 25°}$ = 10.099
sinA = 16sin25^0/10.099, so
A (obtuse) = (180 - 42.0)^0 = 138.0^0 ,
C = (180 – 25 -138.0)^0 = 17.0^0
Hence answers to 1 d.p. are
b = 10.1, A= 138.0^0 , C =17.0^0
[n.b >1 d.p. precision should be used for b in calculating the angles.]

Qn.8 (a) –5**i** + 2**j** (b) $\sqrt{29}$ = 5.4 (to 1 d.p.), direction is at an anti-clockwise angle from the zero line of 158.2^0 (to 1 d.p.).
(2.8 radians (to 1 d.p.) would also be a correct answer.)

Qn.9 (a) $f'(x) = -6x^2 + 18x = -6x(x-3)$ which = 0 when x =0 or x=3.
(b) $f''(x) = -12x + 18$ which is
>0 for x=0, so a minimum
<0 for x=3, so a maximum
(c) Value at mathematical minimum = $f(0) = -10$. Then after the maximum at x=3, the curve turns continuously downwards achieving its lowest value in [-1,5] at x=5, namely -35.

Qn.10 (a) $f'(t) = \frac{2}{t} + \frac{18}{t^4}$
const. multiple and sum rules
(b) $-\frac{1}{7}\cos(7x) + 8\sqrt{x} + c$
const. multiple and sum rules

Qn.11 (a) Use v^2-2as = v_0^2-2as$_0$ (see Chapter C2, p.35) with a=-10, v_0=30, s_0=0.
(b) Max ht. is attained when v=0 so s = 45 m.

Qn.12 Integrate : $y = \frac{1}{5}e^{5t} + c$
y = 1 when t = 0 means c=4/5, so solution is
$y = \frac{1}{5}(e^{5t}+4)$

Qn.13 (a) There are 4 ways, viz, (1,4), (2,3), (3,2), (4,1), out of 36 possibilities so probability =1/9
(b) 1/6
(c) $(5/6)^4$ =0.482
(d) 1 – 0.482 = 0.518
(e) mean number of rolls is 1/(1/6) = 6

Qn.14 (a) Mean = 175.3, median = 176
Lower, upper quartiles =173, 178, range =13
inter-quartile range = 5.
[n.b. the more conventional definition of Q1which you may find in sources outside MST121 is the value of the 11/4th individual, which leads to values:
Q1 = 172.5, Q3 = 178.25, IQR = 5.75]

(b) Key points of boxplot are
168, 173, 176, 178, 181

(c) The distribution is left-skewed because the boxplot has a longer tail and a bigger median to quartile distance at the left.

Qn.15 The test statistic of 2.41 is greater than 1.96, so reject the null hypothesis at the 5% significance level, and conclude that the mean numbers of words per sentence used by Patricia Cornwell is significantly greater than that for Kathy Reichs.

Qn.16 (a) x=8, y= 2.836, so predicted no of accidents = 284.

Qn.17 (a) Solve $36+(y-7)^2=100$, i.e. $(y-7)^2=64$ so $y-7=\pm 8$, y=-1 or 15, so the points are (2,-1) and (2,15).
(b) (i) centres are (-4, 7) and (8 ,-2)
distance between centres = $\sqrt{144+81}$ = 15
(ii) Radii are 10 and 5, sum = 15.
(iii) they are touching each other.
(c) (i) a = 1, b = 2
(ii) sketch a parallel curve displaced 2 units to the right and going through (3,0) and (11,2).
(iii) $(x, y) \mapsto (x+2, y)$

Qn.18
(a) 2003 – 1987 = 16. 16th. root of 14 is 1.17932, so r = 0.179 to 3 d.p..
(b) $P_n = 1{,}000(1.179)^n \quad n=0,1,2\ldots$
(c) After 8 yrs $1000(1.17932)^8$ = 3,740 approx. (3,730 using r = 1.179)
(d) Solve $40{,}000 = 1{,}000(1.179)^n$ i.e. n = log40 / log 1.179 = 22.4 , so population first exceeds in year 23, that is 2010.
(e) No, because factors such as over-fishing, and unsustainable demand on food supplies are likely to alter the underlying assumptions.
(f) excess = 14,000 times 0.179 = about 2500.

Qn.19 (2003 students)
(a) Write $u=8+x^2$

$\frac{dp}{du}=\frac{1}{2\sqrt{u}}$, $\frac{du}{dx}=2x$, $\frac{dp}{dx}=\frac{dp}{du}.\frac{du}{dx}$ hence result.

(b) $q=xp$ so $\frac{d}{dx}(xp)=p\frac{d(x)}{dx}+x\frac{dp}{dx}$

$$= p + x\frac{dp}{dx} = \frac{(8+x^2)+x^2}{\sqrt{8+x^2}} = \frac{2(4+x^2)}{\sqrt{8+x^2}}$$

(c) $r=x/p$ and so

$$\frac{dr}{dx}=\frac{1}{p^2}\left(p\frac{d(x)}{dx}-x\frac{dp}{dx}\right)=\frac{1}{8+x^2}\left(\sqrt{8+x^2}-\frac{x^2}{\sqrt{8+x^2}}\right)$$

$$= \frac{(8+x^2)-x^2}{(8+x^2)\sqrt{8+x^2}} = 8.(8+x^2)^{-3/2}$$

(d) $f(x)$ cuts x axis at x=1 and x=4, and is positive only between these values.
$f(x)=10p'(x)-q'(x)$ so integral of $f(x)$ is $10p(x) - q(x) = (10-x)\sqrt{8+x^2}$ between limits 1 and 4 which is $6\sqrt{24}-27$ = 2.39 to 2 d.p.

Qn.19 (2002 students)
(a) Solve $8 = 4e^{0.03t}$, i.e. 0.03t=ln2, i.e. t = ln2 / 0.03 = 23.1 years.

(b) $\frac{dP}{dt}=0.12e^{0.03t}=0.03P$, so proportionate growth rate is 0.03.

(c) Curve is exponential until at t=100, P=80.3
Thereafter p continues to grow but curvature is reversed and graph tends to its limiting value of 200 from below

(d) Population reaches 150 million when $50 = 884e^{-0.02t}$, i.e. t = - ln(50/884) / 0.02 = 143.6 years.

(e) Integral $=\frac{1}{50}\left[200t+(50\times 884)e^{-0.02t}\right]_{100}^{150}$

= {(600 + 44.0) – (400 + 119.6)} = 124.4
so average population size = 124.4 millions.

Qn.20 (a) (i) the empirical distribution is left-skewed.
(ii) draw a curve which is a rough continuous outline of the frequency distribution.
(iii) estimate is proportion of total area under curve drawn in (ii) which lies to the left of a vertical line through x=330.

(b) (i) conf int. = $331.75 \pm 1.96\frac{5.81}{\sqrt{50}}$
= (330.1, 333.4)
(ii) He can conclude that the mean volume in his cans almost certainly exceeds the legal requirement.
(iii) (a) Sample should be four times larger because standard deviation of sample means is inversely proportional to the square root of sample size.
(b) No, because each sample has a different standard deviation, and chance variation determines whether any particular sample standard deviation is greater or less than the norm.